Interval-Class Exchanges in a Two-Dimensional Pitch-Class Space

Mitchell Turner

Composers and theorists have turned to issues regarding a unified musical space countless times when writing or analyzing music. Schoenberg (1975: 220) writes that "THE TWO-OR-MORE-DIMENSIONAL SPACE IN WHICH MUSICAL IDEAS ARE PRESENTED IS A UNIT."[1] This paper takes the view that there must be some level of unity between the linear and vertical pitch-class space in order for the piece's pitch structure to cohere. Unity manifests when an interval class (ic) is expressed in the horizontal and vertical pitch-class spaces, creating a two-dimensional pitch-class structure in which significant intervallic similarities exist between the dimensions. An ic, first expressed vertically, conceptually rotates 90° such that simultaneity becomes line, resulting in a horizontal instance of an initial vertical ic. The ic is thus "exchanged" between the vertical and horizontal dimensions. These linear/vertical *interval-class exchanges* are the focus of this paper.

Models of Voice Leading

A great deal of recent research has focused specifically on the problematic issue of atonal voice leading.[2] Joseph N. Straus (1997: 237-238) has identified three models for atonal voice leading: prolongational, transformational, and associational.[3]

---

[1] Capitalization is original. See also Schoenberg 1995: 13-15 and 1978: 28.

[2] Recent studies include Baker 1990; Chapman 1978; Forte 1988, 1992; Jurkowski 1998; Klumpenhouwer 1991; Lewin 1990, 1998; Morris 1998; O'Donnell 1997; Roeder 1984, 1994; Straus 1997; Turner 1999.

[3] Prolongational analyses of tonal music may be found in Schenker 1979, Salzer 1962, and Forte 1982. Baker 1983, 1990, and Morgan 1976 extend the Schenkerian approach into the atonal repertory. For prolongations based on aural perception see Lerdahl 1983, 1989. For additional bibliography, see Straus 1997.

Prolongational voice-leading analyses attempt to determine pitch significance through the identification of pitch and chord prolongations that are based either on tonal hierarchies or on theories of aural perception.[4] In both, prolongation is defined as an aural phenomenon. Allen Forte, for example, writes that "prolongation refers to the ways in which a musical component... remains in effect without being literally represented at every moment" (Forte 1982: 142). Whereas prolongation may exist for tonal music, its export for atonal analysis poses many problems, since the chords and hierarchies of tonality are not present.[5] Lerdahl (1987) avoids this problem by investigating prolongation with respect to aural perception (i.e., from the point of view of the listener). Theories of aural perception base prolongations on the relative salience of an atonal pitch event. Loudness, metric placement, register, and timbre are among the elements used to establish the relative importance of a pitch event. Total reliance on salience, however, predicates a piece's pitch hierarchy solely on non-pitch factors, which seems potentially problematic.

The transformational approach stems from the research of David Lewin.[6] In it, pc sets connect to subsequent pc sets, which may or may not have the same prime form, uncovering "...a sense of an underlying pitch-class counterpoint, one relatively immune to the vagaries of the contextual surface" (Straus 1997: 242). Analyses are concerned not simply with the chords themselves but with the transposition (T) and inversion (I) operations, or transformations, which connect chords.[7]

---

[4] A cogent critique of the prolongational approach in atonal music is presented in Straus 1987.

[5] Baker (1983 and 1990) attempts to extend tonal notions of prolongation into the atonal repertory. For a critique of this analytical extension, see Straus 1987 and 1997, and also Turner 1999 (especially Chapter 1).

[6] Sources regarding transformational voice-leading analysis include Lewin 1987, 1990, 1994, 1998; Roeder 1984, 1994; Klumpenhouwer 1991; Jurkowski 1998; O'Donnell 1997.

[7] In understanding the basic mechanics of transformational voice leading, imagine a three-note vertical voicing of a C-major chord which is followed by a three-note vertical voicing of a G-major chord. The transformational graph would reveal that the operation T_7 connects the chords and would note how the root of the C chord maps to the root of the G chord, the E maps to B (third to third), and the G maps

Associational analyses seek linear pitch-class sets (pc sets) with striking similarities to the prevailing vertical pc sets. Forte (1988: 324-338) identifies linear instances of the "Tristan chord" in his analysis of the Prelude to *Tristan und Isolde.* Though not stated formally by Forte, his graphs of the Prelude show a high degree of unity between the vertical pc sets and the long-term linear motion, i.e., similarities between harmony and melody.[8] This paper adds to the associational approach by identifying similarities between linear and vertical dyads. I seek to formally describe the associational approach, investigating dyadic interval class replication which unifies the harmony and melody. The discussion is limited to linear/vertical dyads, leaving later investigations to codify linear/vertical pc set similarities of the type shown in Forte 1988.[9]

An Exchange of Interval Class between the Linear and Vertical Dimensions

In Example 1, the vertical dyad (B3,A4)[10] moves first to the vertical dyad (B♭3,A♭4) and second to (A3,G4). Intuitively, one can hear an intervallic connection between two transpositionally-related dyads (see Example 1a). Further, if the transposition level between the two dyads is equal to the interval found in each vertical dyad (Example 1b), not only is this linear/vertical dyadic equality eminently hearable, it creates intriguing analytical implications. While the transposed minor seventh is easily heard in both, Example 1b increases the prominence of ic2, since the horizontal dyads (A4,G4) and (B3,A3) are both ic2.

---

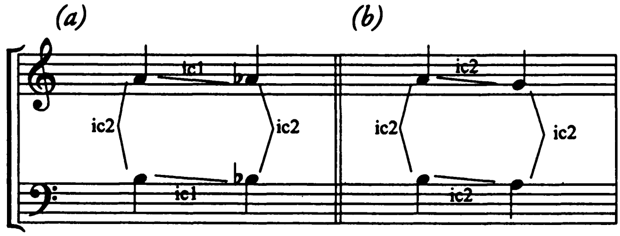
to the D (fifth maps to fifth). With respect to atonal music at least, linear connections, or "voices," may be viewed as more significant than the contextually derived "lines" found in the actual music (Straus 1997: 241-242).

8 For more on the associational approach, see Chapman 1981; Forte 1988, 1992; Turner 1999.

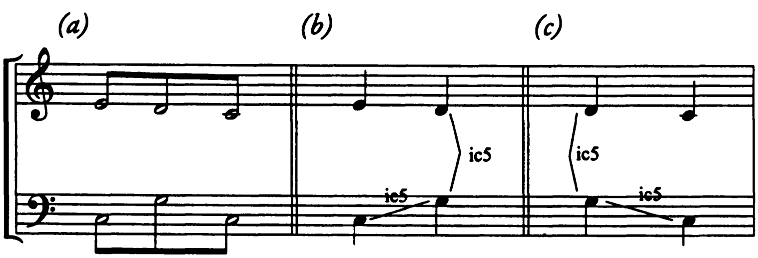
9 Robert Morris (1998: 180-184) has developed a taxonomy of types of dyadic voice leading.

10 References to specific pitches are designated by pitch-class letter and octave number. C4 means middle C; D4 refers to the D just above middle C; C5 is one octave higher than middle C. This system is recommended by the Acoustical Society of America.

Example 1a-b. Transpositionally-related vertical ic2 dyads.



*Example 2a-c. Schenker's fundamental structure (*Ursatz*).*



As in Example 1b, the linear/vertical presence of only a small number of interval classes may elevate that ic's role in the pitch structure. Such situations are the focus of this article. Below, pairs of vertical dyads are classified to show various relationships between the vertical and horizontal interval classes. In each case, there will be a unity between the harmony and melody, demonstrable in both tonal and atonal contexts.

This type of unity is found tacitly in the theories of Heinrich Schenker; in fact, the fundamental structure (*Ursatz*) identified by Schenker has exactly this type of linear/vertical intervallic unity (see Example 2). In Example 2b, the linear/vertical prominence of ic5 is shown between the first two vertical dyads from Example 2a. Example 2c continues this analysis with respect to the second and third vertical dyads from Example 2a. Ic5's significance to both the harmony and melody is great. Schenker's fundamental structure "...unfolds a chord horizontally while the counterpointing lower voice effects an *arpeggiation* of this chord through the upper fifth" (Schenker 1979: 4). His view identifies the triad as a privileged entity. The triad is thus itself an *a priori* structural progenitor and Schenker's fundamental structure shows how the triad is expressed over some period of time. I contend that it is the linear/vertical presence of ic5 which may have implications for analyzing pieces outside of the tonal system.

In this context, any interval may support a structural dyadic motion just as ic5 does in the fundamental structure of Schenker. Schenker's ultimate appeal is to the triad. The ultimate appeal of this study is to interval class between intersecting horizontal and vertical dyads. Since this paper will investigate atonal music, logical pitch structures may be shown in which the triad plays no role. In order to make this possible, the following assumption is made:

A dyad is considered structural if and only if the dyad is presented vertically and horizontally, such that the vertical and horizontal presentations intersect.

No attempt is being made to show prolongation of non-triadic sonorities; rather, coherence and unity between the horizontal and vertical dyads is shown, even in the absence of common practice triadic harmony and its accompanying hierarchies.

Taking the above definition as a point of departure, an ic will occur twice in order to become structural: once vertically and once horizontally. Comparisons between the dyads are made via interval class. As discussed below, each comparison will involve the ics found in two vertical dyads (Vic_1 and Vic_2) and two horizontal dyads (Hic_1 and Hic_2). Structural dyadic motions will be restricted to intersecting dyads in which some equality exists between the ics of one Vic and one Hic, demonstrating the unity between the harmony and melody.

Formal Definitions for Dyads

To systematically explore relationships between two distinct types of dyads (i.e., vertical and horizontal), a notation for their theoretical discussion is devised.[11] This formalization makes the following assumptions for all subsequent examples and discussions:

1. The musical space encompasses the twelve pitch classes under equal temperament.
2. The distance between any two pitch classes is measured in half steps.
3. These measurements of distance fall into one of seven interval classes[12] (ics), such that interval class zero (ic0)[13] accounts for all unisons and octaves, ic1 accounts for all pitches with a distance of 1 or 11 half steps, etc.
4. When making theoretical generalizations, distinct pitches are referred to as P1, P2, etc.
5. Similarly, the variables q, r, s, t, etc., are used to represent any of the twelve pitch classes.

---

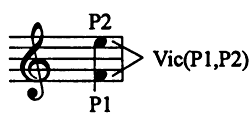
[11] Many of the formal notations are taken from Lewin 1987. I have used Lewin's notation as a point of departure and have freely adapted it for the purposes of the discussion.

[12] I use the term "interval class" as defined by Forte (1973: 14).

[13] The interval class ic0 was omitted "for practical purposes" by Forte (1973: 14). I refer to intervals with half-step distances of 0, 12, 24, etc., as being of the interval class zero (ic0).

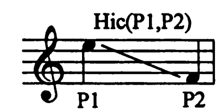
Given two pitches P1 and P2 that occur simultaneously, the distance between P1 and P2 is referred to as a *vertical interval class*, or Vic(P1,P2);[14] thus, in Example 3, Vic(P1,P2) = 1, or ic1. It is important to note that this dyad could be expressed as Vic(P1,P2) or as Vic(P2,P1) since the interval class is identical. For convenience, I put the lower of the two pitches in the first place of the ordered pair in vertical dyads. This notion may be generalized: let P1 and P2 be any pitches, and s and t their respective pitch-classes. The vertical interval class between P1 and P2 is expressed as Vic(P1,P2) = Vic(s,t), evaluated as the interval class between s and t.

Example 3. Vertical interval class notation.



Given two pitches P1 and P2 that occur in succession within the same voice, the distance between P1 and P2 is referred to as the *horizontal interval class*, or Hic(P1,P2); thus, Hic(P1,P2) = 1, or ic1, as shown in Example 4. As a convenience, the pitch that occurs first temporally is placed first in the ordered pair.[15] To generalize this notion, let P1 and P2 be any pitches, and s and t their respective pitch classes. The horizontal interval class between P1 and P2 is expressed as Hic(P1,P2) = Hic(s,t), evaluated as the interval class between s and t.

Example 4. Horizontal interval class notation.



---

[14] Read "the vertical interval class between P1 and P2."

[15] Since Hic(P1,P2) and Hic(P2,P1) equal the same ic, they are considered equivalent.

Dyadic motions involve four pitches, which are grouped into four distinct dyads. This division into dyads is formalized as follows: given any four pitches P1, P2, P3, and P4, if a vertical dyad is formed between P1 and P3 and a vertical dyad is formed between P2 and P4, then there are also two horizontal dyads, (P1,P2) and (P3,P4), as shown in Example 5. The ics of the vertical dyads are Vic(P1,P3) and Vic(P2,P4) and they have two corresponding horizontal dyads whose ics are expressed Hic(P1,P2) and Hic(P3,P4). The following discussions dispense with the cumbersome notation and simply refer to Vic(P1,P3) as Vic_1 and Vic(P2,P4) as Vic_2. Similarly, Hic(P1,P2) and Hic(P3,P4) are called Hic_1 and Hic_2.[16]

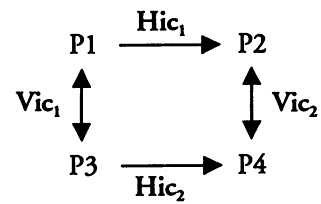
Initial Analyses

Example 6a renders mm. 2-3 from Wagner's *Tristan und Isolde*, in which the soprano voice plays a primary role in unifying the intervals found in the harmony and melody.[17] Example 6a shows two chords marked I and J. The chord labeled I is the so-called "Tristan chord," and the chord marked J is a dominant-seventh chord. Example 6b isolates the soprano and bass voices of I and J from Example 6a, in which Vic(F,G♯) = Hic(G♯,B). This ic equality of linear and vertical dyads satisfies the assumption above. Similarly, Example 6c investigates the ics between the horizontal and vertical dyads created by the soprano and alto voices from chords I and J. Again, ic3 unifies the structure as it is found as Hic(G♯,B) and Vic(D,B), but ic3 is the upper horizontal dyad and the second vertical dyad. Example 6d reveals a voice exchange between the soprano and tenor voices of chords I and J. Based on the assumption above and the particulars of Example 6a-d, ic3 may be viewed as the structural ic of these measures. Other ics are

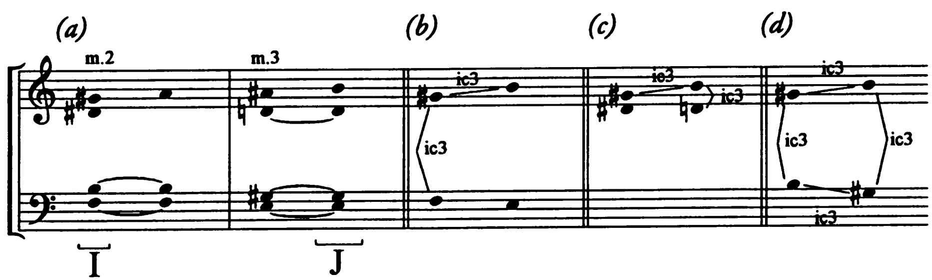
[16] No inference as to primacy or relative importance may be derived from which dyad is called Vic_1. The labeling of the first dyad as Vic_1 and the second dyad as Vic_2 is purely for convenience and up to the analyst. Similarly, either of the upper or lower voices may be labeled Hic_1.

[17] For a "transformational" approach to this section see Klumpenhouwer (1991: 2:6-2:9). See Forte 1988 for an associational view of the pc sets in the Prelude.

Example 5. General musical space of two dyads.



Example 6a-d. Wagner, Tristan und Isolde, *Act I, mm. 2-3.*



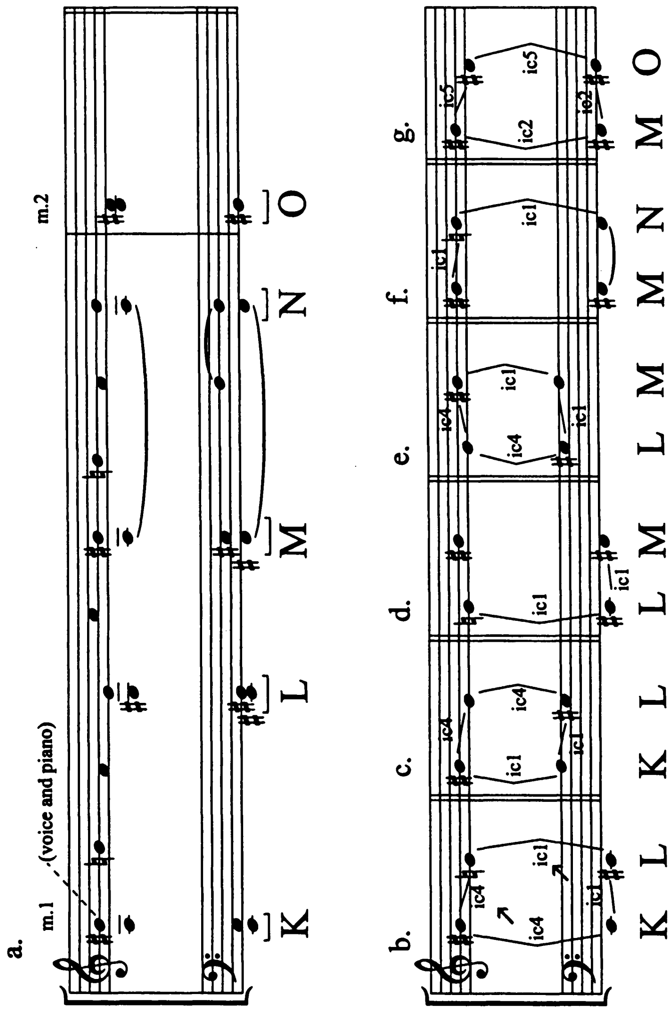
present—e.g., Hic(E,F), Vic(E,B), Vic(D♯,G♯), and Hic(D♯,D)—but they occupy either the horizontal pitch space exclusively or vertical pitch space exclusively and are thus considered non-structural in terms of the theory presented here.

More than one ic may be involved in the pitch structure involving an exchange of interval classes. In this case, the ics of Hic_1 and Vic_1 are the same, as are Hic_2 and Vic_2. I write this informally as: $Hic_1 = Vic_1$ and $Hic_2 = Vic_2$. The structural assumption regarding linear and vertical presentations of the ic is still satisfied. Example 7a-g comes from m. 1 of "Da meine lippen reglos sind," the fourth song in Schoenberg's *Book of Hanging Gardens*, Op. 15. There are four primary voices here, since the piano doubles the vocal line and the alto and "tenor" are initially octave doubling. Five vertical chords are marked K, L, M, N, and O. Example 7b extracts the bass and soprano voices from chords K and L, revealing that $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$. The arrows show how the ic4 and ic1 are exchanged between the vertical and horizontal dimensions. Example 7c shows a similar exchange of ics between the soprano and alto voices from chords K and L. Example 7d-e involve the connections between chords L and M. Between bass and soprano, only ic1 is found vertically and horizontally (Example 7d); however, in Example 7e, the alto and soprano show the exchange of ic4 and ic1 such that $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$. Example 7f (soprano and bass from chords M and N) shows an oblique context in which ic1 is involved in Hic_1 and Vic_2. Example 7g returns to the one to one ic exchange such that $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$.

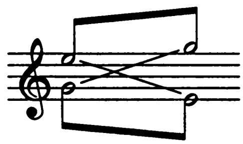
Formally Defining Voice Exchange as an Intervallic Arpeggiation

Voice exchanges found in atonal music are similar to those found in tonal music in that they have structural implications. Whereas pitch is the critical structural factor in tonal voice exchanges, the ics of the horizontal and vertical dyads reveal equally important structural relationships in atonal music. Voice exchange occurs when the upper and lower pitch classes of one vertical dyad are interchanged in a subsequent dyad, such that the upper pitch class in the first vertical dyad becomes the lower pitch class in the

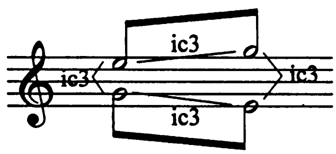
Example 7a-g. Schoenberg, Op. 15, No. 4, mm. 1-2. (a) Reduction of mm. 1-2, with significant verticalities; (b-g) Chord connections show ic1 and ic4 structuring m. 1, and ic2 and ic5 structuring m. 2.



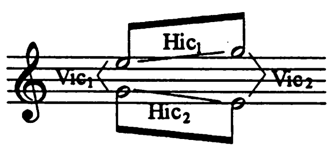
Example 8. Voice exchange.



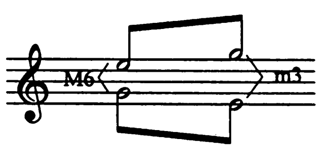
Example 9. Vertical and horizontal ics in voice exchange.



Example 10. Hic and Vic in voice exchange.



Example 11. Vertical dyads in a complementary relationship.



second vertical dyad and the lower pitch class in the first vertical dyad becomes the upper pitch class in the second vertical dyad, as shown in Example 8. This results in the arpeggiation of the first vertical dyad in both horizontal voices. The solid beams in Example 8 show the linear pitches involved in the linear/vertical dyad.

Since each vertical and horizontal dyad involves the same pitch classes, voice exchange shows complete unity of vertical and horizontal interval classes. In other words, a single interval class exerts control over all of the vertical and horizontal intervals, as shown in Example 9. Therefore, an intervallic unity between the dyads in the harmony and those in the melody is established.

The equality between the vertical and horizontal ics holds in all instances of voice exchange. To generalize this notion, the vertical interval class between the first vertical dyad (Vic_1) is equal to the horizontal interval class (Hic_1) between the upper horizontal dyad, or $Vic_1 = Hic_1$, and the vertical interval class between the second vertical dyad (Vic_2) is equal to the horizontal interval class (Hic_2), or $Vic_2 = Hic_2$ (Example 10). Since all of the ics are identical, voice exchange may be represented as $Hic_1 = Vic_1 = Hic_2 = Vic_2$.

One further limit must be expressed to define voice exchange fully: the pitch classes of the upper horizontal voice must be an actual arpeggiation of the first vertical dyad, and the pitch classes of the lower horizontal voice must also be an actual arpeggiation of the first vertical dyad. Thus, the pitch classes of the first vertical dyad will be identical to the pitch classes of the second vertical dyad. This requires that the vertical ordered pitch-class interval of the first dyad (Vi_1) will be in a complementary relationship to the vertical ordered pitch-class interval of the second dyad (Vi_2), as shown in Example 11. Thus, voice exchange occurs when both of the following conditions are met:

1. $Hic_1 = Vic_1 = Hic_2 = Vic_2$.
2. Upper and lower voices are both actual arpeggiations of the first vertical dyad.

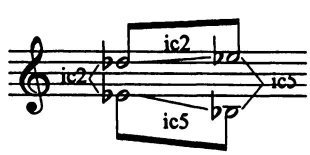
Voice exchange shows a real integration of harmony and melody, whether found in tonal or atonal music. The implications for identifying pitch structures in atonal music are great. Though the importance of voice exchange in tonal music derives from pitch replication within the context of a key, the unity between horizontal and vertical interval classes may be more important for atonal musical analysis.

Formal Definitions of Interval-Class Exchange

Voice exchange is not the only type of dyadic motion to show such intervallic unity. Other related arpeggiations can be imagined that are similar to voice exchange. Whereas in voice exchange, the upper and lower voices literally exchange pitch classes, in an interval-class exchange, the interval classes between vertical and horizontal dyads are exchanged, such that Vic_1 exchanges its ic with Hic_1, and/or Vic_2 with Hic_2. In accordance with the assumption mentioned above, there will be a unity between the harmony and melody, and equalities between the ics of intersecting vertical and horizontal dyads prove crucial. This is shown below as ***interval-class exchange*** (***icex***).

Icex exists in several forms. The primary distinction between them generally involves the appearance of either three or four pitch classes in the four-note collection. Interval-class exchange type I, *icex-I,* refers to instances where three different pitch classes are involved; *icex-II* refers to instances with four different pitch classes.

Example 12. Interval-class exchange type I (icex-I).

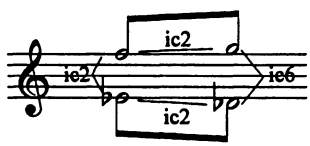


As shown in Example 12, icex-I meets the following condition: $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$. Vic_1 and Vic_2 are not the same but $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$, showing a unity between the horizontal and vertical ics. In other words, Vic_1 and Vic_2 are horizontalized as Hic_1 and Hic_2, respectively. In most instances, icex-I shows the actual arpeggiation of vertical dyads. In Example

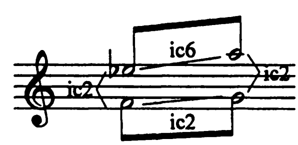
12, Vic_1 consists of the dyad (E♭,D♭), which is also the dyad of the upper horizontal voice. The second vertical dyad, (B♭,E♭), involves the same pitch classes as the lower horizontal dyad. Example 7b, c, e, and g demonstrate several instances of icex-I.

Interval-class exchange type II, *icex-II*, is similar to icex-I and occurs when three of the interval classes are equal. There are two cases. In the first case, the following condition is met: $Vic_1 = Hic_1 = Hic_2$,[18] as shown in Example 13. The second case appears in Example 14, in which the following condition is met: $Vic_1 = Vic_2 = Hic_1$.[19] The prevalence of ic2 shows a high degree of unity between the horizontal and vertical ics in both cases.

Example 13. Icex-II, case 1.



Example 14. Icex-II, case 2.



As mentioned above, a significant distinction between icex-I and icex-II lies in the total number of discrete pitch classes involved. Icex-I generally involves three distinct pitch classes and icex-II usually involves four; however, this aspect of icex is not part of its formal definition. In addition, icex-I exhibits an actual arpeggiation of one of the vertical dyads, which is not the case in icex-II. Note, however, that since any voice exchange satisfies the

[18] Vic_1 can refer to either the first or second vertical dyad.

[19] Hic_1 can refer to either the upper or lower horizontal dyad.

definitions of both icex-I and icex-II, all instances of voice exchange are also instances of icex.[20]

Example 15 shows a special type of icex-II involving ic3, where Vic_1 is a minor third and both upper and lower horizontal voices move outward by a minor third. In this special case, $Vic_1 = Hic_1 = Vic_2 = Hic_2$.[21] It might appear that this special type of icex satisfies the definition of voice exchange. But there is no actual arpeggiation of the first vertical dyad, and thus the definition of voice exchange is not satisfied.[22] Other hybrid cases come to light whenever the intervals involved create symmetrical divisions of the octave (i.e., tritone, fully diminished-seventh chords, or augmented chords).

Formalizing Other Dyadic Motions

Limited icex occurs when only one of the vertical intervals is horizontalized (i.e., $Vic_1 = Hic_1$), as shown in Example 16. The solid beam shows the pitches between which the linear ic4 is found. The dashed beam shows that the other lower linear dyad's ic is not found vertically. This type of motion shows a moderate degree of unity between horizontal and vertical ics, since only one of the Vics is arpeggiated. This motion is quite common. The fundamental structure from Schenker's theory involves a limited icex dyadic motion (see Example 2b and c). Limited icex was prominent in the Wagner example as well, where the soprano voice's horizontal ic3 formed a limited icex relationship with the bass and alto voices (see Example 6b and c).

Parallel motion (Example 17) rarely shows an equality between Hic and Vic. Parallel motion by an ic shown to be important within a musical section, however, may be viewed as significant, as

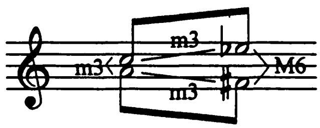
---

[20] Icex-I requires ($Vic_1 = Hic_1$) and ($Vic_2 = Hic_2$), and icex-II requires either ($Vic_1 = Hic_1 = Hic_2$) or ($Vic_1 = Vic_2 = Hic_1$). Voice exchange, defined as $Vic_1 = Hic_1 = Vic_2 = Hic_2$, always satisfies the conditions for both icex-I and icex-II. Thus all voice exchanges are also icex.

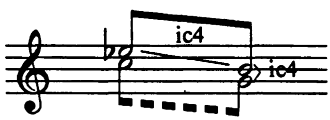
[21] This special case requires that there be an equality between certain intervals (not interval classes), such that $Vi_1 = Hi_1 = Hi_2$.

[22] This holds equally for the complementary form involving a major sixth.

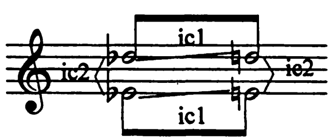
Example 15. Special icex-II involving the minor third.



Example 16. Limited icex.



Example 17. Parallel motion.



shown in the Debussy analysis later in this article. A distinction must be drawn between a parallel motion with no unity between the Hic and Vic and a special type of parallel motion, parallel/icex, that shows the following unity: $Hic_1 = Vic_1 = Hic_2 = Vic_2$. A more thorough discussion of parallel/icex follows.

Parallel/icex, a hybrid dyadic motion, concerns the parallel motion of two vertical dyads by a horizontal ic equal to the ics of both vertical dyads, such that $Hic_1 = Vic_1 = Hic_2 = Vic_2$, as shown in Example 18. Vic_1 and Vic_2 are of the same interval class, as are Hic_1 and Hic_2, and $Hic_1 = Hic_2 = Vic_1 = Vic_2$. This might appear to fit the definition of voice exchange, but fails, as the first vertical dyad is not arpeggiated in both voices.[23] This case does satisfy the definition of both icex-I and icex-II;[24] thus, it is classified as parallel motion with icex. It is not designated as icex-I or icex-II since it has properties common to both. Ic2 was presented in a parallel/icex context in the opening ear-training example that began this study (Example 1b). An example of parallel/icex can be found in Schoenberg's Op.15, No. 6, "Jedem werke bin ich" (Example 19a-b). Looking at the outer voices from the piano part only, the chords marked T and U show ic3 between each of the vertical and horizontal dyads, designating this an instance of parallel/icex.

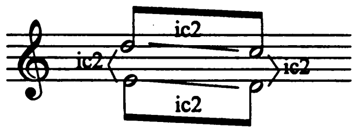
Oblique/ic rotation, a special type of oblique motion, shows some unity between dyads (Example 20).[25] In defining it, I have

[23] There is one exception involving the ic6. Parallel/icex by a tritone satisfies the test of voice exchange, as both horizontal voices do arpeggiate the first vertical dyad; thus tritones that move in parallel motion by tritone will be referred to as instances of voice exchange.

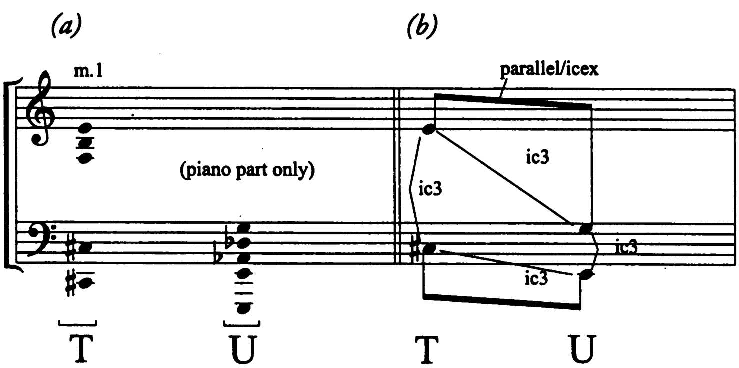
[24] For a motion to be icex-I, $Vic_1 = Hic_1$ and $Vic_2 = Hic_2$ must be true. Since the example shows these equalities to be true, the definition of icex-I is satisfied. But it also satisfies both cases of icex-II. Case 1 requires that $Vic_1 = Hic_1 = Hic_2$. Since Vic_1, Vic_2, Hic_1, and Hic_2 are all ic2, the definition for the first case of icex-II is satisfied. Case 2 requires that $Vic_1 = Hic_1 = Vic_2$. Since Vic_1, Vic_2, Hic_1, and Hic_2 are all ic2, the definition for the second case of icex-II is satisfied; thus, parallel motion with icex is simultaneously an instance of icex-I and icex-II.

[25] I have borrowed the idea of pitch rotation from Bernard 1987. His discussion involves the rotation of two pitches around a third to create a pair of trichords that are intrinsically related. I have adapted this idea of rotation to include the rotation of a single pitch around a fixed pitch. The result of the rotation will create a second dyad that is of the same interval class as the first but with one new pitch class.

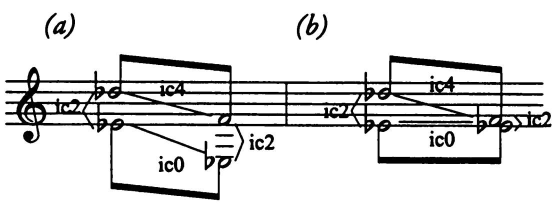
Example 18. Parallel/icex.



Example 19a-b. Schoenberg, Op. 15, No. 6, m.1.

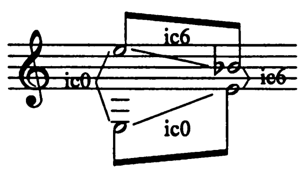


Example 20a-b. Oblique/ic rotation.

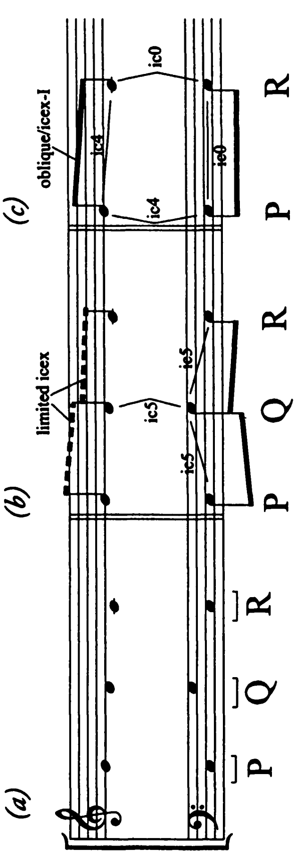


equated the intervals of a perfect octave and a perfect unison (i.e., motion by 12 half steps mod 12 is equivalent to zero half steps), both being represented by ic0. Example 20a shows oblique motion by octave; Example 20b involves oblique dyads where one voice is stationary. In both, there is a clear equality between Vic_1 and Vic_2.

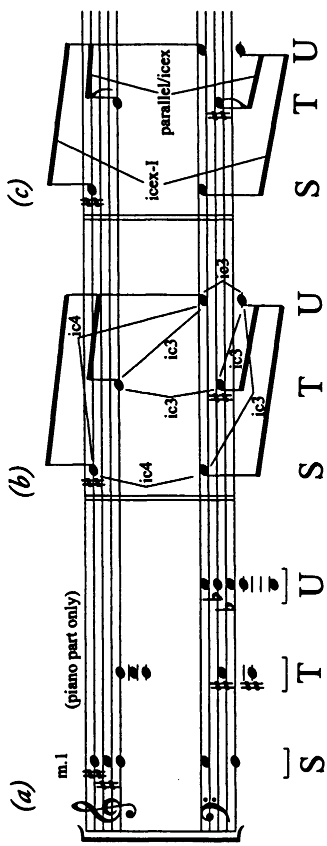
Example 21. Oblique/icex-I.



Another hybrid type of motion blends the motion of icex in two oblique contexts. The first is *oblique/icex-I* which occurs when the perfect octave is presented both vertically and horizontally, as shown in Example 21. Vic_1 and Hic_1 are both ic0, hence $Vic_1 = Hic_1$. Since Hic_1 is ic0, the motion is oblique. $Vic_2 = Hic_2$, because both are ic6. Since the definition of icex-I requires that $Hic_1 = Vic_1$ and that $Hic_2 = Vic_2$, this motion satisfies the definition of icex-I, and thus it is referred to as oblique/icex-I. Looking back at Schenker's fundamental structure, one finds two successive limited icex motions (see Example 22a-c). The vertical dyads, marked P, Q, and R, involve successive elided dyadic motions, which form a kind of dyad chain. Example 22c shows an oblique/icex-I dyadic motion between P and R, suggesting a possibility of reductive analysis using the relative length (or span) of the dyadic motion, with no appeal to the triad. This type of reduction may exist in atonal music as well. Looking at Schoenberg Op.15, No. 6 again, Example 23a-c shows no dyadic relationship between the outer voices of the chords marked S and T and a parallel/icex between the outer voices of chords T and U (recall Example 19b), but there is a dyadic relationship between the outer voices of chords S and U. In Example 23c, a quasi-middle-ground graph shows the dyadic motions found in the outer voices



Example 22a-c. Dyads found in Schenker's fundamental structure (Ursatz).

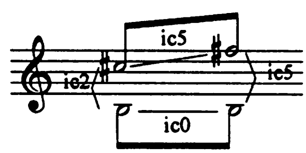


Example 23a-c. Schoenberg, Op. 15, No. 6, m.1.

of the piano part, in which the icex-I is given higher priority than the parallel/icex, owing to its greater span length.[26]

A third hybrid is ***oblique/limited icex***, shown in Example 24. This motion fits the definition of limited icex since $Hic_1 = Vic_1$. The dashed horizontal beam usually found in limited icex is not used here, since in oblique/limited icex the oblique horizontal voice shows a connection by repetition.

Example 24. Oblique/limited icex.



Any dyadic motion that shows intervallic similarities between its Hic and Vic may be useful in atonal analysis. Pitch class similarities may be present; however, interval class establishes a dyadic motion's analytical potential.

Analyses with Dyadic Motions: Examples from Debussy and Schoenberg

The identification of these dyadic motions shows a necessary unity between the ics used in the harmony and melody within a section of music. Analytical attention, therefore, may focus on a small pool of highly important ics, which establish a connection between various dyads.

In the first phrase (mm. 1-5) of Debussy's "Nuages" (from *Nocturnes*), certain dyads appear both harmonically and melodically. The first phrase progresses from (B4,F♯5) to (G4,B4) (see m. 1 and m. 5 respectively), as shown in Example 25.[27] The

---

[26] A view toward "transformational middle-grounds" may be found in Straus 1997, especially Examples 6 through 15.

[27] Doublings in the original are omitted as they do not change the analysis.

discussion following shows how these two vertical dyads are arpeggiated and transposed, unifying the overall section.

Measures 1-5 of "Nuages" break up into four small sections. Each section is a measure long and involves a high level of intervallic arpeggiation. There can be no doubt that ic4 and ic5 are important to the first phrase: 22 of the 25 vertical dyads belong to ic4 or ic5. These two ics are important to the contrapuntal lines as well. In eight of the ten dyadic motions identified in Example 25, either ic4 or ic5 is presented as both a horizontal and a vertical dyad. Clearly, other ics are used in the melodic lines, but only ic4 and ic5 pervade both the harmony and the melody, raising their level of significance.

Limited icex plays the greatest role in the short-term motions. Three different limited icex motions unfold within m. 1. One occurs between the adjacent vertical dyads (B4,F♯5) and (B♭4,D5) and involves the horizontalization of ic4. This limited icex may be represented as Hic(F♯,D) = Vic(B♭,D) = ic4. A second limited icex is found between the vertical dyads (A,E) and (A♯,C♯) and may be expressed as Hic(E,C♯) = Vic(A♯,C♯) = ic3.[28] The third limited icex encompasses the entire measure from vertical (B4,F♯5) to vertical (A♯4,C♯5). This limited icex is an intervallic arpeggiation of the initial vertical (B4,F♯5) and may be represented as Vic(B,F♯) = Hic(F♯,C♯) = ic5. The last vertical dyad in m. 1, (A♯4,C♯5), connects to the vertical (B4,F♯5) at the beginning of m. 2 by limited icex, further reinforcing ic5. At this point, ic5, ic4, and ic3[29] have been expressed both horizontally and vertically. These ics, therefore, are the most important within the first measure. Ic1 and ic2 are not involved in the intervallic arpeggiation of any vertical dyad. In other words, ic1 and ic2 occur only as horizontal dyads, limiting their significance within this section of "Nuages".

All of the vertical dyads in m. 2 are either ic5 or ic4, and these ics are found horizontally in an icex-II and a limited icex motion. The limited icex occurs in the middle of m. 2 between the vertical (B4,F♯5) and (B♭4,D5) dyads. This limited icex, as in m. 1, is established by ic4 since Hic(F♯,D) = Vic(B♭,D) = ic4. The icex-II

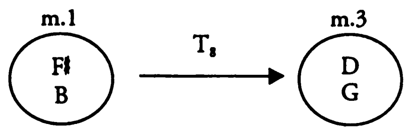
[28] This is the only dyadic motion that involves ic3.

[29] As noted earlier, ic3 only appears in m. 1.

spans the entire measure, from the vertical (B4,F♯5) to the vertical (G♯4,C♯5). This icex-II involves the equality of Hic(F♯,C♯) = Vic(B,F♯) = Vic(G♯,C♯) = ic5. Thus, three different dyads are ic5. The limited icex and icex-II motions in m. 2 solidify the structural role played by ic4 and ic5.

The (G4,D5) that begins m. 3 relates by parallel motion to the (B4,F♯5) from m. 1. This parallel motion is significant since (G,D) is T_8 of the initial (B,F♯) as shown in Example 26.[30] Since T_8 is transposition by a minor sixth and a minor sixth is ic4, these two dyads are related by ic4, a significant transposition level within the context of this section of "Nuages."

Example 26. Transposition of vertical dyad from m. 1 to m. 3.



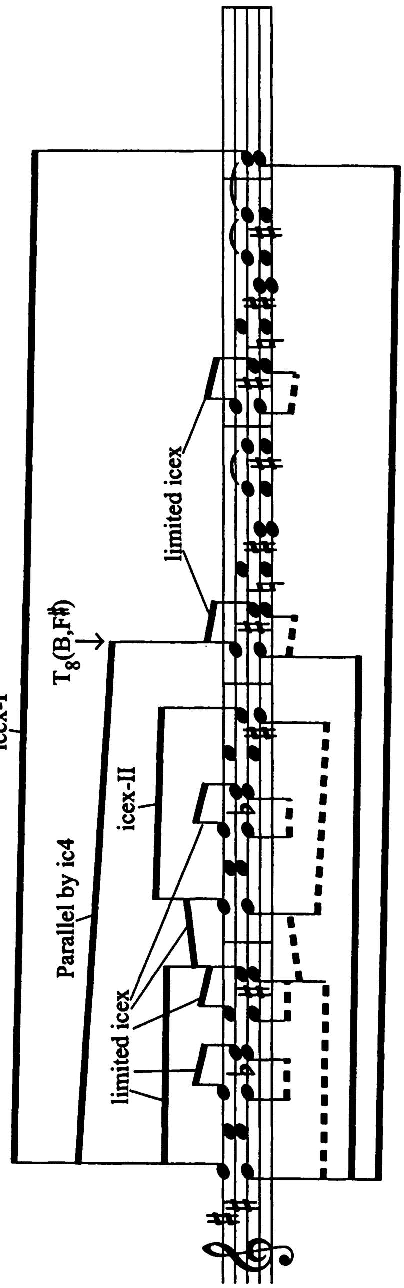
Measures 3 and 4 are identical to one another, each involving one limited icex that occurs at the beginning of each measure between the vertical (G4,D5) and (F♯4,A♯4) dyads. This limited icex raises the significance of ic4 since Hic(D,A♯) = Vic(F♯,A♯) = ic4.

The overall motion from m. 1 to m. 5 exhibits icex-I, where both ic4 and ic5 are found vertically and horizontally. The vertical dyad (B4,F♯5) in m. 1 is expressed horizontally as (F♯5,B4) in the upper voice through m. 1-5. The lower voice arpeggiates a horizontal (B4,G4) dyad over mm. 1-5, and this same dyad appears vertically in m. 5 as (G4,B4). Since Vic(B,F♯) = Hic(F♯,B) = ic5 and Vic(G,B) = Hic(B,G) = ic4, this icex-I strongly suggests that ic4 and ic5 are significant to both the harmony and the melody and may be viewed as the most significant ics within this section of "Nuages."

Throughout mm. 1-5, there is a consistent progression from vertical ic5 to vertical ic4. Two of the limited icex motions in m. 1

---

[30] The nodes and arrows used in Example 19 are adapted from Lewin (1987: 193-219) and Straus (2000: 38-39).



Example 25. Debussy, "Nuages," mm. 1-5.

move from a vertical ic5 to a vertical ic4. The limited icex in m. 2 is similarly constructed. The icex-I encompassing mm. 1-5 also moves from vertical ic5 to vertical ic4.

Investigations of the horizontal and vertical dyads within the first five measures of Debussy's "Nuages" show ic4 and ic5 unifying the harmony and melody. Most of the vertical dyads are ic4 or ic5, though ic3 plays a lesser role. Ic1 and ic2 are present horizontally but are not found as vertical dyads, giving them less analytical significance. Thus, ic4 and ic5 may be viewed as the most important ics within this section of "Nuages."

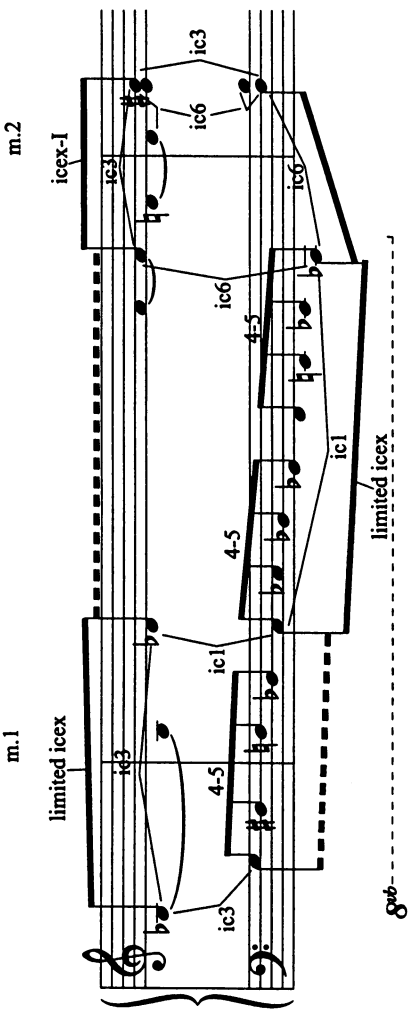
"Als wir hinter dem beblümten tore," song 11 from Schoenberg's Op.15, shows the integration of ic1, ic3, and ic6 into both the harmony and melody of the opening piano gesture. Dyadic motions reveal how this integration occurs in mm. 1-2, which involves a contrapuntal section that culminates in a vertical tetrachord.

The right hand begins with a presentation of (B♭3,D♭4,F4,D4) while the left hand introduces a faster moving series of 4–5[0126] pc-sets (see Example 27). These two melodic lines work together, creating significant dyadic motions. The initial vertical dyad (G2,B♭3) forms a limited icex with the horizontal (B♭3,D♭3) in the upper staff. Since Vic(G,B♭) = Hic(B♭,D♭) = ic3, the initial vertical ic3 may be said to have been arpeggiated in the upper staff. Upon reaching the vertical dyad (C2,D♭4), ic1 comes into play (see m. 1). A second limited icex begins immediately, involving ic1 as Vic(C,D♭) = Hic(C,C♭). The C2 of the vertical (C2,D♭4) dyad initiates the second presentation of the horizontal 4–5, and the C♭1 of the horizontal (C2,C♭1) dyad concludes the third melodic 4–5. C2 and C♭1 are boundary pitches of these two 4–5 pc-sets and help to delineate the set-classes as segments.

The vertical (C♭1,F4) in m. 1 connects to the outer voices of the tetrachord in m. 2 by icex-I.[31] The harmonic and contrapuntal links may be found in the equalities of Vic(C♭,F) = Hic(C♭,F) = ic6 and Hic(F,G♯) = Vic(F,G♯) = ic3.

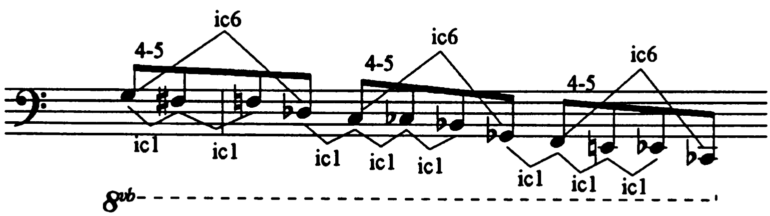
[31] The D4 (m. 1) connects to the tetrachord in m. 2 by ic6 (a significant ic in the tetrachord), but since it does not function in a dyadic motion it will not be discussed further.

Example 27. Schoenberg, Op. 15, No. 11, mm. 1-2.



Ic1 and ic6 are prominent in the linear 4–5 pc sets of the left hand (see Example 28). The boundary interval of each four-note collection is ic6 (e.g., from G2 to D♭2 in the first 4–5). Within each collection, ic1 is found twice; and each four-note collection connects to the next by ic1. Ic4 is also present in the horizontal 4–5; however, ic4's role does not become apparent until the vertical tetrachord in m. 2. Ic1, ic3, and ic6 may be viewed as the most significant ics within this section of Op.15, No. 11.

Example 28. Schoenberg, Op. 15, No. 11, m. 1. Series of 4-5[0126] pc sets.

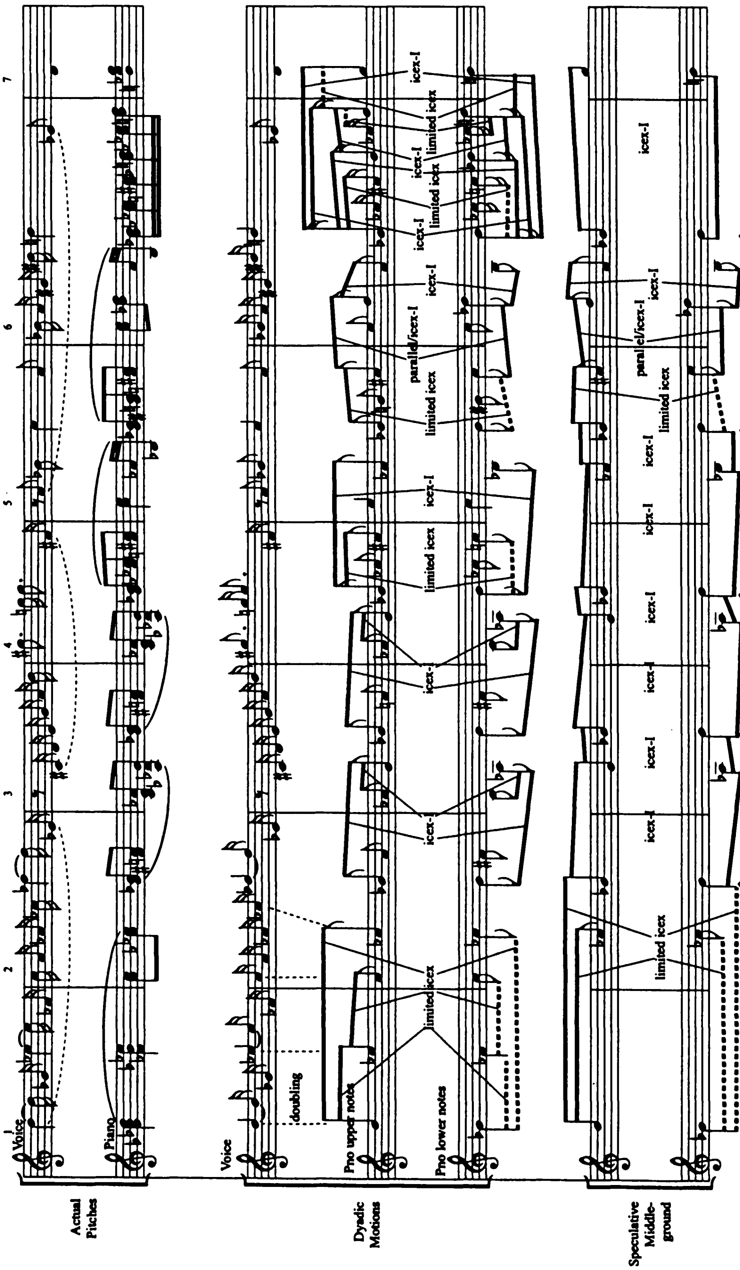


In song 7 ("Angst und hoffen"), mm.1-7 show multiple dyadic motions in the piano accompaniment (see Example 29). The graph details the dyadic motions created by the outer voices of the piano part. The dashed horizontal beams coincide with the phrase markings included by Schoenberg, revealing a structural significance to the phrase markings. In each phrase, the level of intervallic unity is striking. The ics of the harmony and melody consistently produce structural dyadic motions.

A speculative middle-ground graph (Example 29) also appears. Like Example 23c, the beamed quarter notes are seen as having greater structural weight than the beamed and flagged eight notes. This middle-ground graph is interesting in that it reveals that there is an unbroken chain of dyadic motions from m. 1 to m. 6. In other words, a significant and continuous unity exists between the pitches of the harmony and those of the melody.

Analytical reductions of atonal music may be possible by identifying such continuous chains of dyadic motions. If so, then reductive analysis may be achieved in atonal music, without

Example 29. Schoenberg, Op. 15, No. 7, mm. 1-7.



appealing solely to the "salience conditions" described by Lerdahl (1987). By avoiding the singular use of salience conditions, analytical reductions of pitch may be made based, as they should, on pitch.

If one understands a chain of dyadic motions as a pc set, then the structural interchange of ics may be extended to include pc sets. Just as any ic may be expressed in a linear/vertical manner, so too a linear/vertical pc set may have structural implications. In such a reading, an initial vertical pc set may be expressed melodically, elevating that pc set's role in the piece's pitch structure. Future research will attempt just such an extension. The necessary condition for achieving a unified pitch structure, which would be the basis for reductive analysis, is tied directly to the dual expression of the ic, or pc set, as both a melody and a simultaneity.

References

Baker, James M. 1983. "Schenkerian Analysis in Post-Tonal Music." In *Aspects of Schenkerian Theory*, ed. David Beach. New Haven: Yale University Press.

Baker, James M. 1990. "Voice Leading in Post-Tonal Music: Suggestions for Extending Schenker's Theory." *Music Analysis* 9: 177-200.

Bernard, Jonathan W. 1987. *The Music of Edgard Varèse.* New Haven: Yale University Press.

Chapman, Alan. 1978. "A Theory of Harmonic Structure for Nontonal Music." Ph.D. diss., Yale University.

Chapman, Alan. 1981. "Some Intervallic Aspects of Pitch-Class Set Relations." *Journal of Music Theory* 25: 275-290.

Forte, Allen, and Gilbert, Steven E. 1982. *Introduction to Schenkerian Analysis.* New York: W.W. Norton.

Forte, Allen. 1973. *The Structure of Atonal Music.* New Haven: Yale University Press.

Forte, Allen. 1988. "New Approaches to the Linear Analysis of Music." *Journal of the American Musicological Society* 41: 315-348.

Forte, Allen. 1992. "Concepts of Linearity in Schoenberg's Atonal Music: A Study of the Opus 15 Song Cycle." *Journal of Music Theory* 36: 285-382.

Jurkowski, Edward Martin. 1998. "A Theory of Harmonic Structure and Voice Leading for Atonal Music." Ph.D. diss., Eastman School of Music.

Klumpenhouwer, Henry James. 1991. "A Generalized Model of Voice Leading for Atonal Music." Ph.D. diss., Harvard University.

Lerdahl, F. and Jackendoff, R. 1983. *A Generative Theory of Tonal Music.* Cambridge: MIT Press.

Lerdahl, Fred. 1989. "Atonal Prolongational Structure." *Contemporary Music Review*: 65-89.

Lewin, David. 1987. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press.

Lewin, David. 1990. "Klumpenhouwer Networks and Some Isographies that Involve Them." *Music Theory Spectrum* 12/1: 85-120.

Lewin, David. 1994. "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Opus 11, No. 2." *Journal of Music Theory* 38: 79-101.

Lewin, David. 1998. "Some Ideas About Voice Leading." *Journal of Music Theory* 42/1: 15-72.

Morgan, Robert P. 1976. "Dissonant Prolongation: Theoretical and Compositional Precedents." *Journal of Music Theory* 20: 49-91.

Morris, Robert. 1998. "Voice-Leading Spaces." *Music Theory Spectrum* 20/2: 175-208.

O'Donnell, Shaugn J. 1997. "Transformational Voice Leading in Atonal Music." Ph.D. diss., City University of New York.

Roeder, John Barlow. 1984. "A Theory of Voice Leading for Atonal Music." Ph.D. diss., Yale University.

Roeder, John Barlow. 1994. "Voice Leading as Transformation" in *Musical Transformation and Musical Intuition: Eleven Essays in Honor of David Lewin*, ed. Raphael Atlas and Michael Cherlin. Roxbury, MA: Ovenbird Press, 41-58.

Salzer, Felix. 1962. *Structural Hearing: Tonal Coherence in Music*. New York: Dover.

Schenker, Heinrich. 1979. *New Musical Theories and Fantasies. Vol. 3: Free Composition*. Translated by Ernest Oster. New York: Schirmer Books.

Schoenberg, Arnold. 1975. *Style and Idea*, ed. Leonard Stein. Berkeley, CA: University of California Press.

Schoenberg, Arnold. 1978. *Theory of Harmony*. Translated by Roy E. Carter. Berkeley, CA: University of California Press.

Schoenberg, Arnold. 1995. *The Musical Idea and the Logic, Technique, and Art of Its Presentation*, ed. Patricia Carpenter and Severine Neff. New York: Columbia University Press.

Straus, Joseph N. 1987. "The Problem of Prolongation in Post-Tonal Music." *Journal of Music Theory* 31: 1-21.

Straus, Joseph N. 1997. "Voice Leading in Atonal Music." In *Music Theory in Concept and Practice*, ed. James M. Baker, David W. Beach, and Jonathan W. Bernard. Rochester, NY: University of Rochester Press.

Straus, Joseph N. 2000. *Introduction to Post-Tonal Theory*. 2nd ed. Upper Saddle River, NJ: Prentice Hall.

Turner, Mitchell M. 1999. "Toward a General Theory of Pitch Structure: Unity Between Horizontal and Vertical Pitch-Class Sets." Ph.D. diss., University of Georgia.