Some Unusual Transformations in Bartók's "Minor Seconds, Major Sevenths"

Edward Gollin

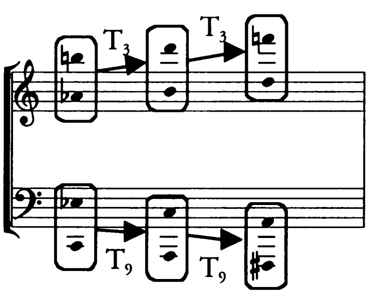
This paper discusses a group of operations that are not the ordinary transposition or inversion operations familiar to many set-theoretical analyses. The paper shows how these operations can be applied to make some analytical observations about pitch-class transformations in a work by Béla Bartók, his "Minor Seconds, Major Sevenths," from the sixth volume of *Mikrokosmos*. Further, the paper demonstrates how members of this particular group can participate in generalized Klumpenhouwer networks (K-nets), and how these analytical tools can be used to reveal recursive structure at deeper levels in the piece.

Instead of simply presenting the operations in the abstract, let us look first at an analytical example that illustrates why these operations can be so useful. Example 1 presents the first eleven measures of No. 7 from Bartók's *Improvisations* op. 20, and reveals a familiar analytical problem: the three circled tetrachords, which punctuate the folk melody, do not all share membership in the same set class. Tetrachords 1 and 3 are both members of set class [0347], but the second tetrachord is a member of class [0235]. Consequently, no transposition or inversion operations can describe the progression of the three successive tetrachords.

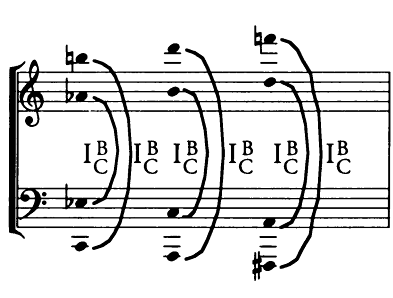
Example 2 shows two possibilities for interpreting the passage using traditional analytical methods. The first possibility, depicted in Example 2a, treats the chordal punctuations as dyad pairs in the left and right hands. In this view the right-hand dyads map successively by T_3, whereas the left-hand dyads maps via T_9. The analysis, however, lacks economy: two transformations are required to describe each step of the progression. Further, the analysis does not strictly describe relations among tetrachords, but rather relations among dyads.

Example 1. Bartók, Improvisations *op. 20, no. 7, mm. 1–11.*[1]

Example 2a.
Transposition of dyad pairs.



Example 2b.
Inversional symmetry.

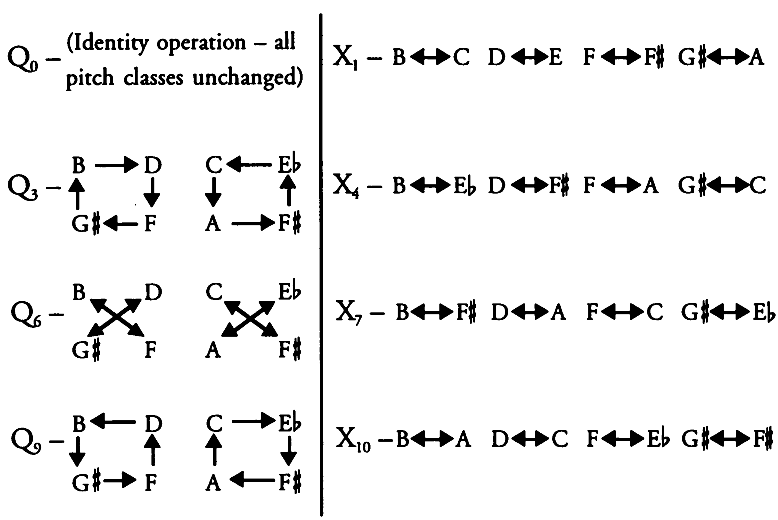


[1] Examples from Béla Bartók's *Improvisations*, op. 20 (© Copyright 1922 by Hawkes and Son (London) Ltd. Copyright renewed) and *Mikrokosmos* (© Copyright 1940 by Hawkes and Son (London) Ltd. Copyright Renewed. Definitive corrected edition © Copyright 1987 by Hawkes and Son (London) Ltd.) are reprinted by permission of Boosey and Hawkes, Inc.

A second possible view of the passage is presented in Example 2b. In this view, the passage coheres because a single inversional axis is exhibited by all three chords: the upper dyads of each tetrachord map to the lower dyads by inversion about the B/C axis. The mirror symmetry of the chords is certainly an important feature of the passage, but the analysis views the "progression" as a static, rather than a dynamic phenomenon.

What would be better is a single operation that can map the pitch classes of tetrachord 1 to those of tetrachord 2. David Lewin presents a germane family of such operations in Appendix B of his *Generalized Musical Intervals and Transformations.*[2] Figure 1 presents the operations of Lewin's group in graphic form. The group consists of eight operations that map pitch classes of an octatonic collection onto one another in a unique way. Note that the circled tetrachords from Example 1 exhaust the pitch classes of an octatonic collection, {B, C, D, E♭, F, F♯, G♯, and A}, and I have used those pitch classes to illustrate the

Figure 1. The Q/X operations.



[2]New Haven: Yale University Press, 1987, pp. 251–253. Lewin refers to the group as STRANS2.

operations of Lewin's group.

Half of the operations consist of "queer" rotations (adopting Lewin's symbology, these are labeled Q): these operations partition the octatonic collection into two [0369]s which rotate in opposite directions by 0, 3, 6 or 9 semitones, following the arrows on Figure 1. For example, under Q3, members of the {B, D, F, G♯} tetrachord rotate 3 semitones clockwise, mapping B to D and so on, whereas members of the {C, E♭, F♯, A} tetrachord rotate 3 semitones counterclockwise.

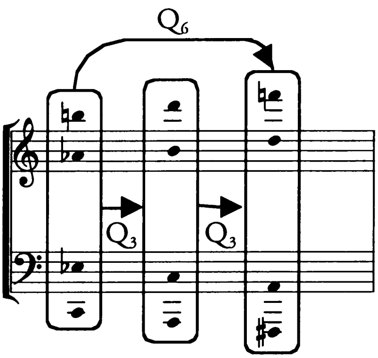
The other four operations are "exchanges" (Lewin labels these with X's) which map each pc from one [0369] to its partner in the other [0369] 1, 4, 7, or 10 semitones away. X1, for instance, exchanges B with C and vice versa, D and E♭, and so on.[3] These 8 operations constitute what I have termed the "octatonic Q/X group," since they operate on the pitch classes of an octatonic collection.[4]

[3]The reader will note several minor discrepancies between Lewin's labels for the operations of STRANS2 and mine. I have used the labels X_{10} and X_7 respectively for Lewin's X_2 and X_5, since it allows a simpler representation of group combination for these operations, as we shall soon see. I use Q_0 and Q_6 for Lewin's R_0 and R_6 respectively. Lewin adopts the latter from his STRANS1 operations on an octatonic set, where they correspond to T_0 and T_6 in 12-tone space. Although Q_6 is equivalent to R_6/T_6 (they have the same ultimate effect), they operate in conceptually different ways: R_6 operates by rotating pcs of both "diminished-seventh" chords in a positive direction by 6 semitones, Q_6 rotates pcs of opposite "diminished-seventh" chords by 6 semitones in opposite directions.

[4]In an unpublished typescript (*Some Notes on Group Theory*, n.d.) Lewin explores an analogous hexatonic Q/X group of six operations, one which partitions a hexatonic collection (i.e. set class [014589]) into two "augmented triads," and shows how certain of the operations describe serial reorderings in Schoenberg's *Ode to Napoleon*. In "Generalized Interval Systems for Babbitt's Lists and Schoenberg's String Trio" (*Music Theory Spectrum* 17/1 (1995): 81–118), Lewin uses operations of a chromatic Q/X group, one which partitions the aggregate into two whole-tone hexachords, to analyze Schoenberg's String Trio.

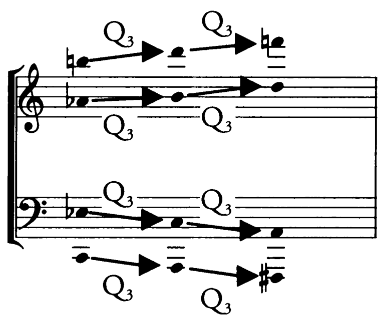
Robert Morris, in "Set Groups, Complementation, and Mappings Among Pitch-Class Sets," (*Journal of Music Theory* 26/1 (1982): 101–144) and "Pitch-Class Complementation and its Generalizations," (*Journal of Music Theory* 34/2 (1990): 175–245), discusses operations of a chromatic "Q/X" group as members

Example 3a.
Analysis by Q/X group operations.



set class: [0347] [0235] [0347]

Example 3b.
Transformational and registral voice leading.

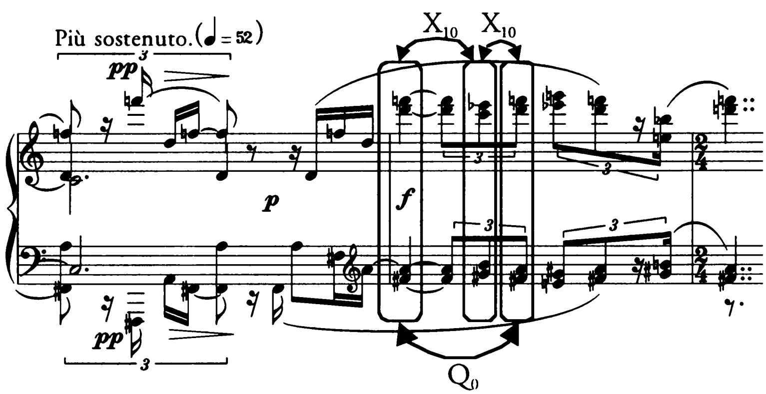


Example 3a analyzes the progression of Example 1 using operations of the octatonic Q/X group. A single operation, Q_3, governs motions between successive tetrachords, and Q_6 governs the progression overall, from first to last tetrachord. Example 3b illustrates how the transformational voice leading of the passage coincides with its registral voice leading. In the highest registral voice, B5 of the first chord maps 3 semitones up to D6 of the second via Q_3; in the lowest registral voice, C2 of the first chord maps 3 semitones down to A1 of the second via Q_3, and similarly for the other voices. The example highlights the fact that—unlike T or I transformations—a Q or an X may transform one pitch-

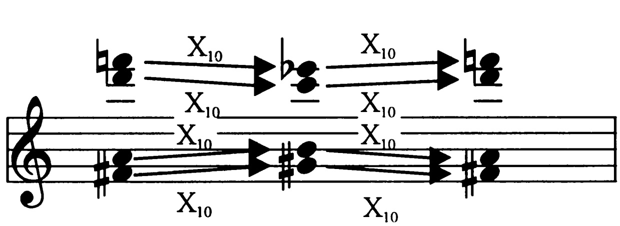
of his set-group ALPHA, placing emphasis not on a whole-tone hexachordal partition of the aggregate, but instead on partitions of the aggregate into six ic 1 dyads. Morris's AL_1 operation comprises the 2-cycles that exchange members within dyads of one such partition: (01)(23)(45)(67)(89)(AB). AL_2 comprises the 2-cycles that exchange members within dyads of the only other ic 1 partition: (12)(34)(56)(78)(9A)(B0). Exhaustive composition of AL_1 and AL_2 generates a group isomorphic to the chromatic Q/X group (Lewin's X_1 and X_e in "Generalized Interval Systems" correspond to Morris's AL_1 and AL_2 respectively). Morris's AL_1 and AL_2 suggest an interesting recasting into mod 8 octatonic step space: ${}^{mod8}AL_1$ and ${}^{mod8}AL_2$, comprising the 2-cycles of step-ic 1 partitions (01)(23)(45)(67) and (12)(34)(56)(70) respectively, would correspond to X_1 and X_{10} in the present work.

class set into another that does not share membership in the same T_n/T_nI set class: of the three Q-related chords in Example 3b, two are in set class [0347] and one is in set class [0235].

Example 4a. Bartók, Improvisations *op. 20, no. 7, mm. 12–14.*



Example 4b. Transformational and registral voice leading.



Example 4a presents mm. 12–13 from the same *Improvisation*, a continuation of the passage from Example 1. The first three tetrachords of m. 13 are circled; arrows indicate X_{10} relations between successive chords. Example 4b illustrates the coincidence of the transformational and registral voice leading.

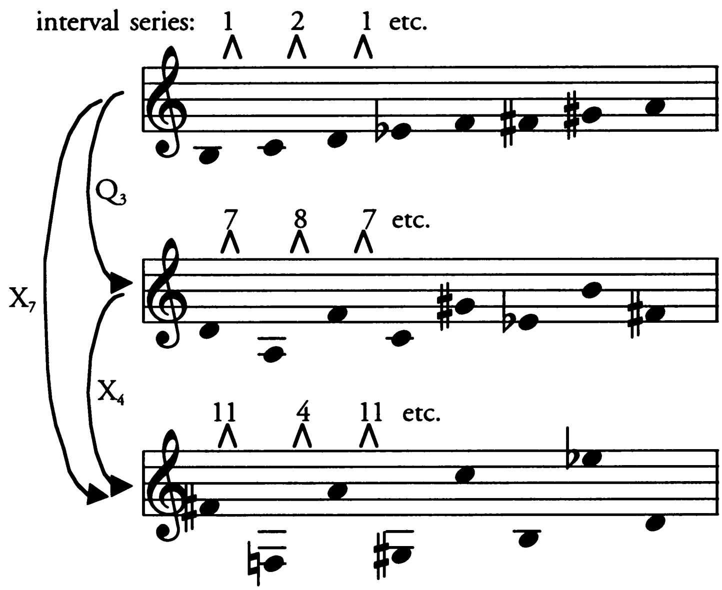
The medial and extreme tetrachords in Example 4 are identical to the first and third tetrachords respectively of Example 3. Yet whereas Example 3 posits a Q_6 relation between its first and third tetrachords, Example 4 posits an X_{10} relation among the medial and extreme tetrachords. Analytically, this degeneracy is not problematic. First, the transformations within the four registral voices support the differently interpreted harmonic transformations among the chords. Second, the distribution of pitch classes in the left and right hands support the different readings: in Example 3, the left and right hands maintain the Q-type [0369] partitions of the octatonic collection, whereas in Example 4, pitch material from both [0369]'s is exchanged *within* each hand.

The Q/X operations are not interval-class preserving in the way T or I transformations are. The Q/X operations preserve interval classes *within* the [0369] tetrachords: ic 3, ic 6 and (trivially) ic 0. We observed in Examples 3 and 4, for instance, how Q3 and X10 preserved ic 3 dyads in the left and right hands parts of the respective *Improvisations* passages. However, under certain of the Q/X operations, interval classes *between* the constituent [0369] tetrachords of the octatonic collection become transformed: ic 1 is exchanged with ic 5; ic 2 is exchanged with ic 4.[5]

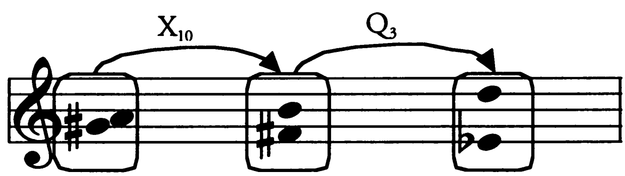
[5]The interval-class-transforming properties of the Q/X operations bear more than coincidental similarity to the familiar multiplicative transformations in 12-tone chromatic space (i.e. the circle-of-fifth transformations). The interested reader can show that Q_3 for example, is equivalent to the 2-transpose of the ${}^{\mathrm{mod8}}M_5$ transformation in mod 8 octatonic step space (let 0, 1, 2, ..., 7 represent the pcs B, C, C♯, ... , A; note that T_2 in octatonic space is equivalent to T_3 in 12-tone chromatic space). Q_9 may similarly be shown to be the ${}^{\mathrm{mod8}}T_6$ of ${}^{\mathrm{mod8}}M_5$ in octatonic step space. Because transposition is interval preserving, the interval-class transformations of ${}^{\mathrm{mod8}}M_5$ are exhibited by Q_3 and Q_9.

The conception of Q_3 and Q_9 as derivatives of a ${}^{\mathrm{mod8}}M_5$ transformation—a transformation in which the even "whole-tone" collection [0246] is invariant and the odd "whole-tone" collection [1357] is transposed by a tritone (= 4 in mod 8 space)—suggests interesting extensions along the lines of Andrew Mead's O_Z operations in 12-tone space. O_Z (for z = 2, 4, 6, 8, t) fixes the even whole-tone hexachord of an aggregate and transposes the odd whole-tone hexachord by 2, 4, 6, 8, or 10 semitones respectively. ${}^{\mathrm{mod8}}O_2$, ${}^{\mathrm{mod8}}O_4$ (= ${}^{\mathrm{mod8}}M_5$), and ${}^{\mathrm{mod8}}O_6$, would analogously fix the even "whole-tone" collection of octatonic step space, while transposing the odd "whole-tone" collection by 2, 4, or 6 steps respectively. Mead discusses O_Z in "Some Implications of the Pitch-Class/Order-Number Isomorphism Inherent in the Twelve-Tone System: Part Two: The Mallalieu Complex: Its Extensions and Related Rows," *Perspectives*

Example 5a. Q/X operations on an octatonic row.



Example 5b. Q/X transformations of dyads.



Example 5a illustrates various Q and X operations acting upon an octatonic row. Under Q_3, the ordered interval series 1, 2, 1, etc. is transformed into the series 7, 8, 7, etc. Semitone adjacencies become "perfect fifth" adjacencies; whole-tone

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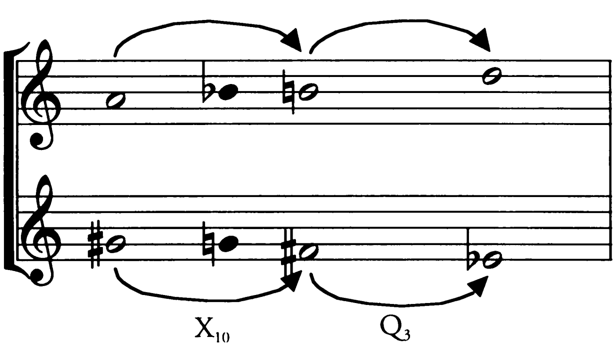
of New Music 27/1 (Winter 1989): 180–233. Mead's O_Z operations are equivalent to Morris's AL_3 operations in "Set Groups," cf. fn. 3.

adjacencies become "minor sixth" adjacencies. Further application of X_4 transforms the interval series 7, 8, 7 into 11, 4, 11, etc. The "perfect fifth" adjacencies of the second series become transformed into "major sevenths", and so on. Example 5b illustrates a progression among dyads, mapped by various Q and X transformations. A dyad expressing ic 1 is mapped via X_{10} to a dyad expressing ic 5, and that dyad is mapped via Q_3 to a different dyad expressing ic 1.

The potential for symmetrical voice leading and the interval-transforming properties—specifically the ability to exchange ic 1 and ic 5—inherent in the Q/X operations, make these operations particularly suitable to describe events in Bartók's "Minor Seconds, Major Sevenths."

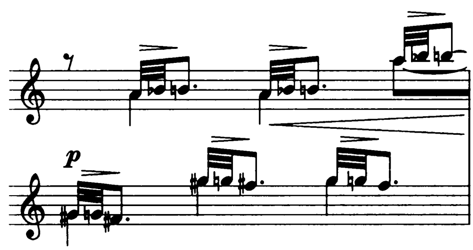
Example 6a. Bartók's "Minor Seconds, Major Sevenths," mm. 1–2.

Example 6b. Reduction.

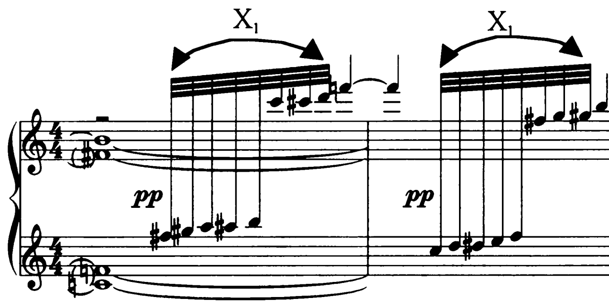


Example 6a presents the opening two measures of the work. Example 6b gives a reduction that shows how the operations X_{10} and Q_3 govern a progression from the inner semitonal dyad, G♯–A, to the two successively expanding registral boundary dyads, F♯–B and E♭–D. The analysis of G and B♭ as non-structural appoggiaturas in *this* context, is fortified by Bartók's later variation of the same figure in m. 37, shown here as Example 7. The scalewise filling of interval spans is a prominent feature of the work, especially within the boundary-interval of the major seventh, as for instance at mm. 21 and 22, shown in Example 8.

Example 7. "Minor Seconds, Major Sevenths," m. 37.

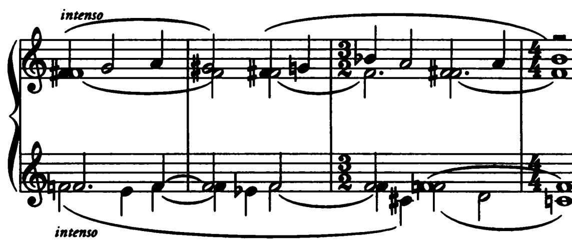


Example 8. "Minor Seconds, Major Sevenths," mm. 21–22.

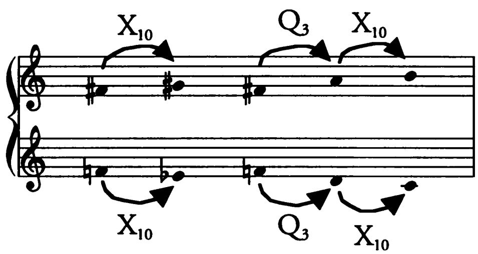


Example 9 presents the *intenso* passage just prior to Example 8, in mm. 18–21. The passage can be understood as a composing-out or variation on the opening wedge figure, now with more chromatic passing and neighboring tones, and with rhythmic displacement. It approaches its final major seventh in two stages: a first stage expands from an initial F–F♯ semitone to a "perfect fourth," E♭–G♯, via X10; a second stage re-expands from the F–F♯ semitone through a fifth, D–A, to a final C–B major seventh, via Q_3 then X_{10}.

Example 9a. "Minor Seconds, Major Sevenths," mm. 18–21.

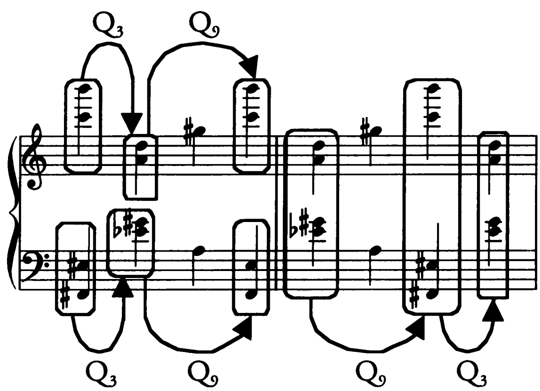


Example 9b. Reduction.



An even more striking example of the Q operations governing progression at the musical surface is found in Example 10, which presents mm. 53–54. The circles I have drawn for m. 53 illustrate how Q_3 and Q_9 transform dyads in each hand. The circles of m. 54 show how these transformations also map the successive [0167] vertical tetrachords of the total texture. Just as we saw in the *Improvisation* (e.g. in Examples 3 and 4), the Q-transformational voice leading of Example 10 is coincident with the registral voice leading. Note how in the circled dyads of m. 53, the Q_3 and Q_9 transformations map dyads of interval class 5 to those of interval class 1. Consequently, these operations do more than simply label successions; they also describe actions that must be executed by the performer of the work. The operations are manifested in the extension and contraction of one's hands, just as a transposition operation might be manifested by the translation of a pianist's hand up or down the keyboard.

Example 10. "Minor Seconds, Major Sevenths," mm. 53–54.



Having observed some musical examples of how the Q/X operations work, we can now investigate some of the pertinent math a bit more closely.[6]

[6]The following brief discussion of certain basic group-theoretical concepts can be augmented by any number of texts on group theory or abstract algebra: for example, Dummit and Foote, *Abstract Algebra* (Englewood Cliffs, N.J.: Prentice Hall, 1991). Writings that explore the relation of group theory and music include (among many others) Lewin, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987) and Robert Morris, *Composition with Pitch Classes* (New Haven: Yale University Press, 1987). Writings on the group-theoretical structure of triadic relations include Richard Cohn, "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," (*Journal of Music Theory*, 41/1 (1997): 1–66); Brian Hyer, "Reimag(in)ing Riemann," (*Journal of Music Theory*, 39/1 (1995): 101–138); Henry Klumpenhouwer, "Some Remarks on the Use of Riemann Transformations." (*Music Theory Online* 0/9 (1994)); and Lewin, "A Formal

Definition 1. A GROUP, (G, *) is a collection of elements, G, and a rule for their combination, *, that satisfy the following conditions:

a) for any g, h in G, g * h is in G

b) G must contain an identity element, e, such that for any g in G, e * g = g * e = g

c) every element g in G must have an inverse g' in G such that g * g' = g' * g = e

d) for any g, h, k in G, (g * h) * k = g * (h * k).

A simple example of a group is the integers mod 12 under addition. The sum of any two mod 12 integers is also a mod 12 integer, the element zero is the identity, and every mod 12 integer n has an additive inverse (i.e. a "negative") 12–n. The twelve ordinary transpositions acting on pcs in the twelve-tone

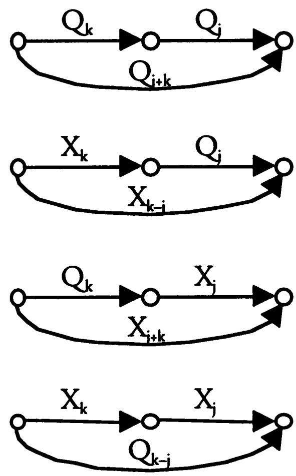
Figure 2. Composition of Q and X operations.

(a) $Q_jQ_k = Q_{j+k}$

(b) $Q_jX_k = X_{k-j}$

(c) $X_jQ_k = X_{j+k}$

(d) $X_jX_k = Q_{k-j}$



Theory of Generalized Tonal Functions," (*Journal of Music Theory* 26/1 (1982): 23–60). Some writings that explore permutation groups in music include Daniel Harrison, "Some Properties of Triple Counterpoint and Their Influence on Compositions by J. S. Bach," (*Journal of Music Theory* 32/1 (1988): 23–50), and John Roeder, "A Geometric Representation of Pitch-Class Series," (*Perspectives of New Music* 25 (1987): 362–409).

chromatic universe are a musically familiar example of another group. Here, the elements are not "things" like numbers or pitch classes, but rather twelve *functions* that operate on pcs. The rule for combination is therefore composition of functions (i.e. first apply A then apply B). Given any T_n and T_m, T_{n+m} is also in the group; T_0 is the identity and the inverse of T_n is T_{12-n}.

Our Q/X operations form a group in the mathematical sense: all Q and X operations compose to yield another Q or X within the group; Q0 is the identity element; Q_3 and Q_9 are inverses, Q_6 is its own inverse, and all X's self invert. Figure 2 shows how the Q and X operations compose with one another, always remaining within the group (i.e. resulting in either some Q or some X).

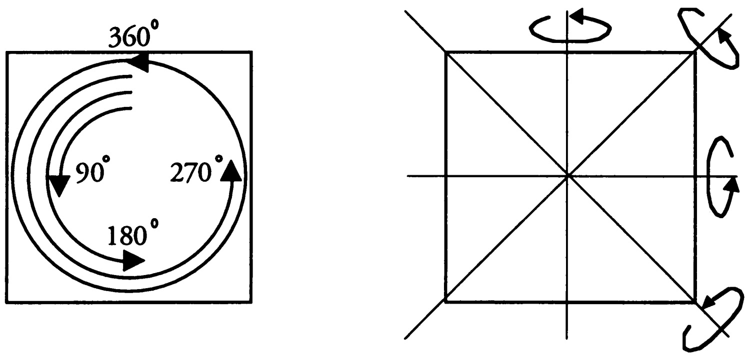
Definition 2. An ISOMORPHISM is a bijective (1-to-1 and onto) function, F, mapping a group of operations, (G, *), onto another group, (J,◊), such that F(g*h) = F(g) ◊ F(h), where g, h are in G. Two groups are said to be ISOMORPHIC if an isomorphism can map one to the other.

In our examples above, the group of mod 12 integers under addition is isomorphic to the group of 12 transpositions under composition. An example of an isomorphism that maps one to the other is simply the function that takes a mod 12 integer n to the transposition Tn. Note that the rules for combination are different in the two groups: addition of numbers is different from composition of functions. What is crucial is that the combination of elements in one group is equivalent to the combination of their isomorphic images in the other. This condition is met in our example [e.g. 2 + 3 = 5 (mod 12), F(2+3) = F(2)F(3) = T_2T_3 = T_5, etc.].

Definition 3. A DIHEDRAL GROUP, D_n, is a group that is isomorphic to the n rotations and n reflections that preserve the symmetry of a regular n-gon.

Figure 3 illustrates the operations of D_4, the four rotations and four reflections that preserve the symmetry of a square. But "dihedral" can refer to any group that has a similar structure—similar in that their respective elements combine in similar ways (a property ensured by the isomorphism). In particular, rotation-type operations always combine additively to yield other rotation type operations. A rotation followed by or following a reflection always yields some reflection, and a reflection following a reflection always yields some rotation. This is not the only distinctive feature of dihedral groups, but it is one which will be important for us later.

Figure 3. The eight operations of the dihedral group D_4 that preserve the symmetry of a square.



4 Rotations 4 Reflections

The familiar T/I operations form a dihedral group, D_{12}. Its 12 transpositions are analogous to the 12 rotations of a regular dodecagon and its 12 inversions are analogous to the 12 reflections [the interested reader may prove this is so]. Our Q/X group is also a dihedral group, in this case D_4: the four Q operations are analogous to the four rotations that preserve the symmetry of a square, the four X operations are analogous to the four reflections.

We will return to this idea as we turn our attention from consideration of Q/X relations between pitch classes and pitch-class sets, and to look at the possibilities of deeper relations among pitch-class networks whose arrows are labeled by Q/X operations. In order to do so, we will need to generalize the notion of a K-net. Since K-nets have received detailed consideration elsewhere, the present discussion will only briefly present some examples of K-nets as they have traditionally been employed, highlighting some concepts that will be useful to us later.[7]

* * *

A K-net is an *interpretation* of a pitch-class set: it highlights particular transformational relationships among the pitch classes of a collection, modeling one way—among many possible ways—of hearing that collection. Example 11 reproduces measures 25–37 of "Minor Seconds, Major Sevenths"; in it I have circled and numbered chords labeled (1) through (7). The chords are neither all of the same set class, nor even of the same cardinality, but they all share one very audible feature: they all express at least two dyads of the semitonal dyad class. In chords (1) through (5), these are manifest as major sevenths, in chords (6) and (7), as literal semitones. Figure 4 interprets each chord as a K-net. The transformational arrows reflect how I hear the chords—arrows labeled T_{11} reflect how I hear the major sevenths of chords (1)–(5), arrows labeled T_1 reflect how I hear the literal semitones of chords (6) and (7), and in the context of the mirror symmetry of the piece thus far, it seems natural to be aware of the inversional relationship between the innermost and outermost voices of the chords.

All seven graphs exhibit a similarity known as "isography." Every pair of graphs shares the same configuration of nodes and arrows, and there is a special function called an "automorphism" that can map the operations labeling arrows of one graph to the operations labeling the corresponding arrows of the other.

[7]More extensive discussion of Klumpenhouwer Networks can be found in Henry Klumpenhouwer, *A Generalized Model of Voice Leading for Atonal Music*, (Ph.D. diss., Harvard University, 1991); also *idem*, "Aspects of Harmony and Row Structure in Martino's Impromptu No. 6," (*Perspectives of New Music* 29/2 (1991): 318–55); also David Lewin, "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Opus 11, No. 2," (*Journal of Music Theory* 38/1 (1994): 79–101); also *idem*, "Klumpenhouwer Networks and Some Isographies that Involve Them," (*Music Theory Spectrum* 12/1 (1990): 83–120).

Example 11. "Minor Seconds, Major Sevenths," mm. 25–37.

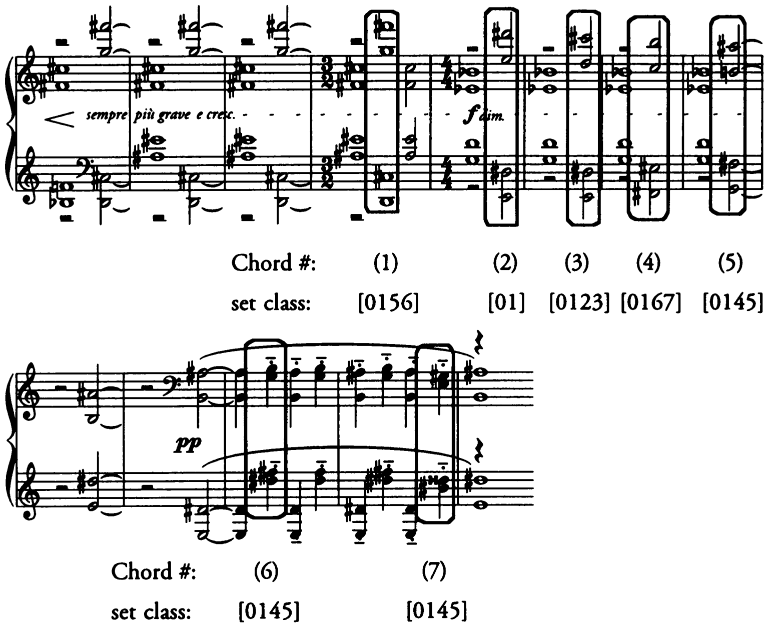
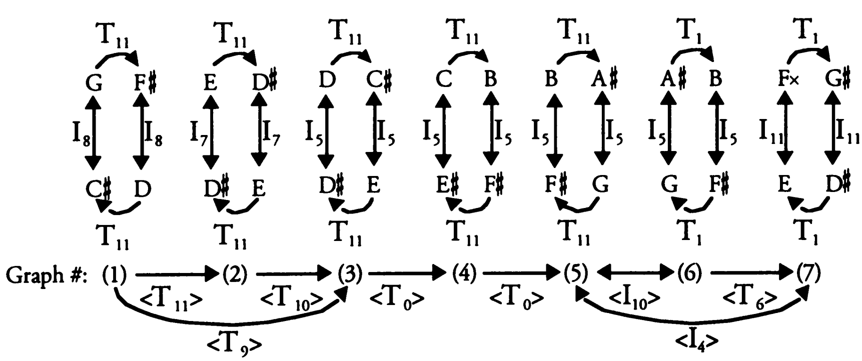


Figure 4. K-net interpretation of chords in mm. 25–37.



Definition 4. An AUTOMORPHISM is a bijective function, F, mapping a group of operations, (G,*), onto itself, such that F(g*h) = F(g) * F(h), where g, h are in G.[8]

The definition stipulates that the structure of a group, the way its elements compose, is preserved by its automorphisms. If one considers a family of isographic networks to constitute a "network class," then the relation between isographic networks and their automorphisms is analogous to the traditional relation between pc-set classes and the T/I operations. Just as a transposition or inversion operation can relate two distinct pc sets that belong to the same set class, an automorphism can relate two distinct transformational networks that belong to the same "network class."

Graphs that have identical corresponding T-labels, such as graphs (1)–(5), are called "positively isographic."[9] The automorphisms that relate positively isographic K-nets are labeled $<T_j>$ (read "Hyper-T-sub-j").[10] Inspection of Figure 4 shows that under $<T_j>$, regular T-subscripts are unchanged and I-subscripts increase-by-j. In our set-class/network-class analogy, positively isographic networks would correspond to the T_n-types induced by transposition alone in the pc-set domain.[11] Another kind of isography, called "negative isography," obtains between graphs with inverted T-labels on corresponding arrows, as between our graphs (5) and (6), or (5) and (7). The automorphisms relating negatively isographic K-nets are labeled $<I_k>$. $<I_k>$ inverts regular T-subscripts, and maps regular I operations whose indices sum to

[8]Lewin provides an extensive formal discussion of the automorphisms of the T/I group in "Klumpenhouwer Networks and the Isographies that Involve Them."

[9]This terminology is adopted from Lewin, "A Tutorial on Klumpenhouwer Networks," p. 88*ff.*

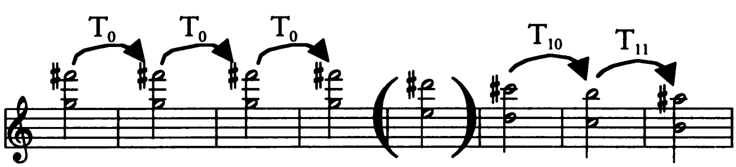
[10]The <T> notation is taken from Klumpenhouwer, *A Generalized Model of Voice Leading for Atonal Music*. Lewin adopts it in "A Tutorial on Klumpenhouwer Networks." Throughout this paper, angle brackets will be used to distinguish automorphisms from pc transformations.

[11]T_n-types are discussed in chapter 4 of John Rahn, *Basic Atonal Theory* (New York: Schirmer, 1980).

k. In our set-class/network-class analogy, negatively isographic networks would correspond to T_nI-related members of a set class.

$\langle T_j \rangle$ and $\langle I_k \rangle$ automorphisms compose the same way ordinary T and I operations do. For instance, on Figure 4, $\langle T_9 \rangle$ between graphs (1) and (3) is equivalent to the composition of $\langle T_{11} \rangle$ and $\langle T_{10} \rangle$. Consequently, the group of $\langle T_j \rangle$ and $\langle I_k \rangle$ automorphisms is isomorphic to the T/I group itself. Be aware, however, that the corresponding elements of these two groups are different operations. T and I map pitch classes; $\langle T_j \rangle$ and $\langle I_k \rangle$ are *automorphisms*, which map T and I operations themselves. In Figure 4, I have not asserted transformational relationships among pitch-class collections. Rather, I am asserting a relation among particular transformational relationships within those collections.

Example 12. Transpositional relationship of dyads in mm. 25–32.



But because the $\langle T_j \rangle$ and $\langle I_k \rangle$ automorphisms of the T/I group are isomorphic to the T/I group itself, a transformational pathway among K-nets has the potential to enter isomorphic relationships with transformational pathways in the pitch-class domain of a work. I show a simple instance of this in Example 12, which presents the upper dyads that fall on the third quarter of each bar in mm. 25–32. The relations among graphs (1)–(5) of Figure 4 are somewhat suggestive of the transpositional relationship, in a temporally retrograde fashion, among the upper dyads of the very chords whose networks are graphed. Comparing Example 12 to Figure 4, we note how the $\langle T_0 \rangle$ automorphisms that obtained among graphs (3)–(5) of Figure 4 are mirrored by the T_0 relations among dyads in mm. 25–28, and that $\langle T_{11} \rangle$ and $\langle T_{10} \rangle$ between graphs (1), (2) and (3) are reflected by the T_{10} and T_{11} mappings to the penultimate and ultimate dyads respectively.

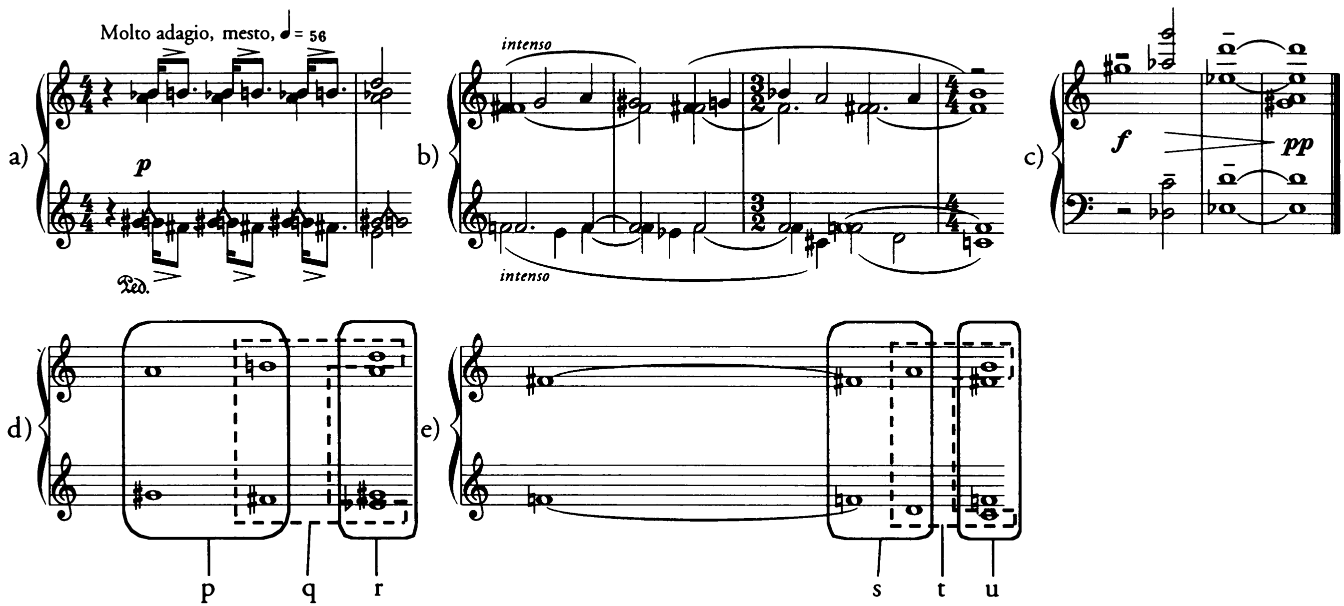
Although Example 12 only explores isomorphic relations among $<T_j>$ automorphisms and ordinary transpositions, relations need not be limited to these. By embracing $<I_k>$ as well as $<T_j>$ automorphisms, one can realize the recursive potential of K-nets. In particular, an interpretation of the transformational relations within a pitch-class collection may be projected onto the deeper structure as a transformational network among several K-nets.

It is not my intention to pursue such an analysis using the T and I labeled K-nets of Figure 4. I merely wanted to introduce some concepts: K-nets; positive and negative isography; a group of automorphisms, each of which maps a group onto itself.

Presently, we shall explore how the Q/X operations discussed earlier may participate in structures analogous to K-nets, and how, by invoking the automorphisms of the Q/X group, we can reveal an interesting example of recursive structure in "Minor Seconds, Major Sevenths."

Example 13a reproduces a passage from "Minor Seconds, Major Sevenths" which we have seen previously, the opening wedge figure of m. 1 to its cadence at the downbeat of m. 2. Example 13b reproduces the *intenso* passage from m. 18 to its cadence at the downbeat of m. 21. Example 13c presents the final three measures of the piece. The passage in Example 13a is preoccupied with the opening wedge motive. The motive culminates in the structural sonority of m. 2, {E♭, G♯, A, D}, a form of set class [0167]. The pcs of this m. 2 sonority are the same as those which conclude the work at m. 71, shown in Example 13c. Beginning in m. 8, the wedge motive occurs at new and various pitch levels. The *intenso* passage at m. 18 presents what I earlier called a composing-out of the wedge motive. The culminating sonority in m. 21, {C, F, F♯, B}, is also a form of set class [0167]—a form which is the unique octatonic complement of the [0167] in m. 2. Through a further development of the *intenso* passage in mm. 23–24, the music arrives at a new sonority in m. 25, {B♭, F, F♯, C♯}, one which retains a common F–F♯

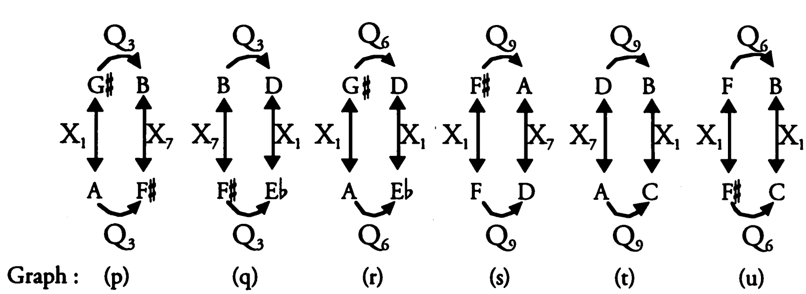
Example 13. "Minor Seconds, Major Sevenths," (a) mm. 1–2; (b) mm. 18–21; (c) mm. 69–71; (d) reduction of (a); (e) reduction of (b).



dyad, but which is not of the octatonic world that preceded. I hear the subsequent music exploring several hexatonic realms, but containing an intervening octatonic reminiscence of the opening (m. 37). A third and longer return of the *intenso* (mm. 43–50) serves as a bridge back to the octatonic world, announced by the arrival of the {F♯, E♯, C, B} sonority at m. 51 (a rearrangement of the m. 21 collection from which we departed). If one views the m. 2 sonority, {E♭, G♯, A, D}, as thesis, and its octatonic complement at m. 21, {C, F, F♯, B}, as antithesis, then the ensuing passage of mm. 53–55 (see Example 10) climactically presents the entire octatonic collection as a dialectical synthesis. The final ten measures of the piece provide harmonic closure, reiterating the {E♭, G♯, A, D} collection of the opening. Although this is not an all-encompassing view of the work, it is an outline which will afford us an adequate frame of reference.

Examples 13d and 13e reduce Examples 13a and 13b respectively to a "structural framework" (analogously to Examples 6b and 9b, we omit "appoggiaturas" and "passing tones"). On Example 13d, I have circled and labeled three four-note collections: the leftmost circle labeled (p) collects the first four notes; the dotted outline labeled (q) collects the final pair of outer voices; the rightmost circle labeled (r) identifies the final simultaneity. Three four-note collections are similarly circled and labeled on Example 13e: the leftmost circle labeled (s) collects the first four notes; the dotted outline labeled (t) collects the final pair of outer voices; the rightmost circle labeled (u) identifies the final simultaneity. Figure 5 interprets each of the 6 collections as a K-net, but now with Q's and X's labeling arrows. Note that within each passage, none of the circled pitch-class collections belong to the same Q/X-set class. In other words, no Q or X operation can map the pitch classes of one to the other.

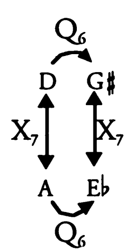
Figure 5. K-net interpretation of four-note collections from Example 13.



A few things are worth mentioning at this point. Although K-nets traditionally have employed only T and I transformations among the pitch classes in a collection, transformations need not be limited to these. However, because there is a unique Q/X mapping from any one pitch class to any other, the Q/X K-nets lack the multiplicity of interpretations afforded by the T/I group. For instance, in the T/I group, given an F♯–G♯ dyad, one might interpret a T_2 relation from F♯ to G♯, or a T_{10} relation from G♯ to F♯, or perhaps one might understand the relation as inversion-about-G. However, in the context of the Q/X group, only a single interpretation obtains, that of X_{10}.

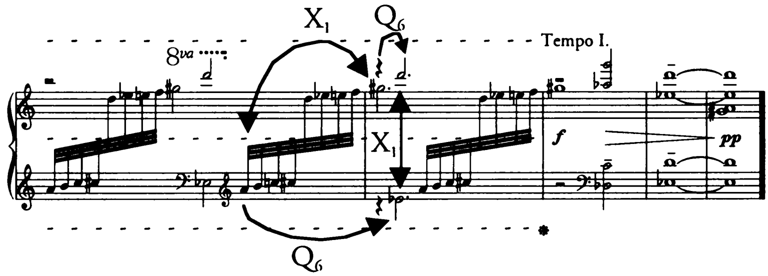
This is not to say that there is no choice in interpretation. For instance, Figure 6 offers another interpretation of the pitch collection of Figure 5, graph (r), which I have labeled (r'). (r') highlights the X_7 relations in the chord, those which one hears as boundary tones of each hand. But as was noted, the pitch-class collection of graph (r) is identical to that of the final sonority in the work. And in light of the closing material, the interpretation of graph (r) in Figure 5 seems more convincing, for it reflects the operations unfolded at mm. 66–67 and prior, as is shown in Example 14. Graph (r) of Figure 5 also asserts the audible prominence of semitone relations within its collection—a feature which comports well with the title of the piece.

Figure 6. Alternative interpretation of a collection r.



(r')

Example 14. "Minor seconds, Major Sevenths," mm. 66–71.



Looking now at all the graphs of Figure 5, we observe that graphs (r) and (u) are identical: they exhibit the relation of equality. Graphs (p), (q), (s) and (t) exhibit a weaker relation among themselves: namely, they are isographic—meaning, as discussed above, that there is some automorphism of the Q/X group that can map graphs to one another, such that the configuration of nodes and arrows on the graph remains unchanged, but the Q/X transformations labeling the arrows do change.

Figure 7 presents the automorphisms of the octatonic Q/X group in graphic form.[12] The eight automorphisms form a group isomorphic to that of the Q/X group itself: both are dihedral groups.

Half of the automorphisms, labeled $\langle Q_j \rangle$, leave Q-subscripts unchanged and rotate the X-subscripts, increasing their value by 0, 3, 6 or 9 units (mod 12). Just as we saw with the $\langle T_j \rangle$ automorphisms discussed above, <Qj> automorphisms map Q/X K-nets that are positively isographic. In the context of our tetrachordal Q/X K-nets, positive isography is exhibited by graphs with identical node-arrow configurations that exhibit identical Q-transformations on corresponding arrows. In Figure 8, the automorphism $\langle Q_6 \rangle$ maps graph (p) to (q) and graph (s) to (t)—in both cases, the Q-subscripts are unchanged and the X-subscripts increase by 6 units mod 12.

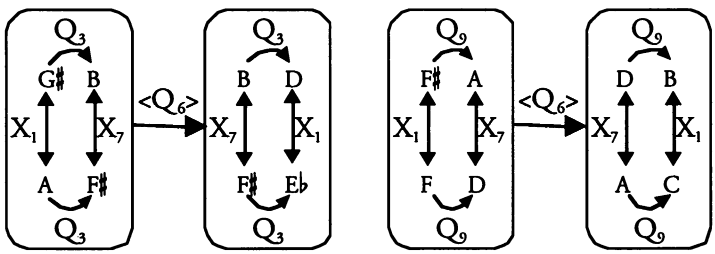
[12]The automorphisms were derived using an algorithm provided in Appendix B of Lewin, "Klumpenhouwer Networks and Some Isographies that Involve Them."

Figure 7. The automorphisms of the Q/X group.

<Q_0> (hyper–Q_0) –	(Identity operation – all operations unchanged)
<Q_3> (hyper-Q_3) –	$\begin{array}{ccc} X_1 & \longleftarrow & X_4 \\ \downarrow & & \uparrow \\ X_{10} & \longrightarrow & X_7 \end{array}$
<Q_6> (hyper-Q_6) –	$X_1 \leftrightarrow X_7$, $X_4 \leftrightarrow X_{10}$
<Q_9> (hyper-Q_9) –	$\begin{array}{ccc} X_1 & \longrightarrow & X_4 \\ \uparrow & & \downarrow \\ X_{10} & \longleftarrow & X_7 \end{array}$

<X_1> (hyper-X_1) –	$Q_3 \leftrightarrow Q_9$	$X_4 \leftrightarrow X_{10}$	(X_1 and X_7 self map)
<X_{10}> (hyper-X_{10}) –	$Q_3 \leftrightarrow Q_9$	$X_1 \leftrightarrow X_4$	$X_7 \leftrightarrow X_{10}$
<X_7> (hyper-X_7) –	$Q_3 \leftrightarrow Q_9$	$X_1 \leftrightarrow X_7$	(X_4 and X_{10} self map)
<X_4> (hyper-X_4) –	Q3$\leftrightarrow$Q9	X1$\leftrightarrow$X10	X7$\leftrightarrow$X4

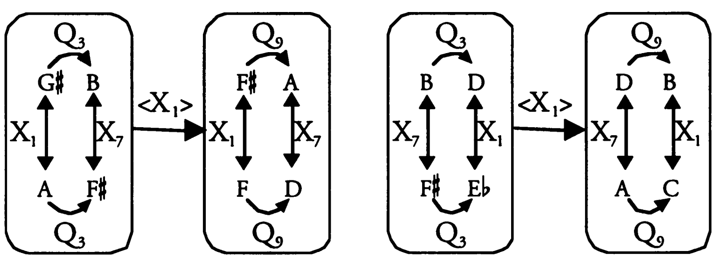
Figure 8. <Q_6> automorphism of tetrachordal K-nets.



Graph : (p) (q) (s) (t)

The other four automorphisms, labeled <X_k>, are analogous to the <I_k> automorphisms of the T/I group. <X_k> automorphisms map Q/X K-nets that are negatively isographic: they map Q operations to their inverses, just as <I_k> map transpositions to their inverses. In Figure 9, <X_1> maps graph (p) to (s) and graph (q) to (t).

Figure 9. <X_1> automorphism of tetrachordal K-nets



Graph : (p) (s) (q) (t)

The <Q_j> automorphisms are similar to regular Q operations in that they combine analogously—they both represent the "rotations" of their respective dihedral groups. Similarly, the <X_k> automorphisms and the X operations are equivalent to the "reflections" in their respective dihedral groups. In other words, the <Q/X> group of automorphisms is isomorphic to the Q/X group itself. Hence transformational relations among Q/X K-nets can be isomorphic to Q/X relations among pitch classes. As was suggested above, the power of isographic Q/X K-nets lies in their ability to reveal recursive structures at surface and deeper levels of a work. Figure 10 illustrates a striking example of recursive networks in "Minor Seconds, Major Sevenths." The network formed from graphs (p), (q), (s), and (t), whose arrows are labeled by the automorphisms of the Q/X group, is itself isomorphic to the networks of graphs (r) and (u), networks whose sonorities we designated as thetic and antithetic in the context of the octatonic unfolding, and cadential in their respective phrases. Thus, the structure of the cadential sonorities is a manifestation, or better, a synthesis, of a higher-order progression toward those sonorities themselves.

Figure 10. Recursive networks in "Minor Seconds, Major Sevenths".